# Second Law Analysis and Optimization of Solar Hot Water Systems

by

Syed Muthee Ulla Shaahib

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

**MASTER OF SCIENCE**

In

**MECHANICAL ENGINEERING**

August, 1989

# SECOND LAW ANALYSIS AND OPTIMIZATION OF SOLAR HOT WATER SYSTEMS

BY

SYED MUTHEE ULLA SHAAHID

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

**MASTER OF SCIENCE**

**In**

MECHANICAL ENGINEERING



AUGUST, 1989

UMI Number: 1381148

*KING FAHD UNIVERSITY OF PETROLEUM & MINERALS*

*DHAHRAN, SAUDI ARABIA*

*This thesis, written by*

***SYED MUTHEE ULLA SHAAHID***

*under the direction of his Thesis Advisor, and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of*

***MASTER OF SCIENCE IN MECHANICAL ENGINEERING***

***Thesis Committee***

Chairman (Dr. Abdul Kerim Kar)

Member (Dr. Al-Sulaiman, F.A.)

Member (Dr. Said, S.A.M.)

Dr. Habeeb Abualhamayel
Department Chairman

Dr. Abdullah S. Al-Zakri
Dean College of Graduate Studies

F1

Date : Nov. 28th 85

College of Graduate Studies
Office of the Dean

*To*

*THE SUN OF HOPE AND THE SOURCE OF GIVING*

*MY MOTHER*

*THE EMBLEM OF SACRIFICE AND THE SYMBOL OF STRONG WILL*

*MY FATHER*

## ACKNOWLEDGEMENT

All praises and glory be to Almighty Allah who blessed me with courage and wisdom to accomplish this research work.

Acknowledgement is due King Fahd University of Petroleum and Minerals for its incredible all out support in undertaking this investigation.

My deep appreciation and thanks are due to my thesis committee chairman and academic advisor Dr. Abdul Kerim Kar. He was generous in time and thought inspite of preoccupations. He enlightened my path to the field of optimization. My thanks are due to other committee members Dr. Al-Sulaiman, F.A. and Dr. Said, S.A.M., for their coordination.

It is my great pleasure to express deep gratitude to Dr. Habib Abualhamayel. Chairman, M.E. Department for his immeasurable cooperation during the course of this research.

I owe sincere thanks to my colleagues and friends especially to Mr. Abdul Aleem and Mr. Mamdouh B. Al-Aidarous who enriched me with moral support, inspiration and motivation.

Last but not the least, special thanks are due to my beloved parents and other family members for their patience, encouragement and continuous prayers which were instrumental in accomplishing this research work.

# TABLE OF CONTENTS

# LIST OF TABLES

## LIST OF FIGURES

# THESIS ABSTRACT

NAME : SYED MUTHEE ULLA SHAAHID

TITLE : SECOND LAW ANALYSIS AND OPTIMIZATION OF SOLAR HOT WATER SYSTEMS

MAJOR : MECHANICAL ENGINEERING

DATE : 26th AUGUST 1989

*Traditionally solar systems are analyzed on the basis of first law of thermodynamics which dictates that energy in a system is not lost, but it is transferred from one form to another. However first law doesn't account for irreversibilities (entropy generation) accompanied during heat transfer across finite temperature difference, fluid friction, mixing of fluids etc. In this investigation solar domestic hot water (SDHW) systems were analyzed from the stand point of second law of thermodynamics. Attention was mainly focussed on analysis and optimization of solar hot water systems for minimum entropy generation while satisfying dynamic, thermal, and total hot water requirement constraints.*

*The SDHW systems were simulated using actual weather data while providing the necessary hot water requirement in accordance with Rand, Vitro and ASHRAE hourly load profiles. Systems were simulated for various tank volume to collector area and total load mass flow over tank volume ratios. In each case entropy generated and auxiliary energy consumed were obtained.*

*For every case simulated, the distribution of load mass flow during 24 hour period was obtained by minimizing total entropy generation during the same period. For each case, the total entropy generated and auxiliary energy consumed were compared with the corresponding simulation results.*

*Optimization of SDHW systems for minimum entropy generation show considerable reduction in entropy generation and auxiliary energy consumption. Optimization studies point out that for a particular system, entropy generation per liter of hot water withdrawal is not a strong function of flow profile. Both simulation and optimization results indicate that for minimum entropy generation and minimum auxiliary energy consumption load should be approximately equal to storage tank size for one day's data and tank size to collector area ratio should be approximately between 50 to 70.*

MASTER OF SCIENCE DEGREE
KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
Dhahran, Saudi Arabia.
1989

ملخص الرسالة المقدمة
من طالب الماجستير :
سيد مطيع الله شاهد
المقدمة إلى قسم الهندسة الميكانيكية
بجامعة الملك فهد للبترول والمعادن بالظهران
بتاريخ ٢٦/٨/١٩٨٩م

## تحليل القانون الثاني للديناميكية الحرارية وتعظيم سخانات المياه الشمسية

إن من العادة تحليل الأنظمة الحرارية التي تستخدم الطاقة الشمسية على اساس القانون الأول للديناميكية الحرارية والذي يقول ان الطاقة لا تفنى ولا تستحدث ولكن تتحرك من شكل الى آخر ؛ وبهذا فإنها لا تأخذ بالاعتبار اللامعكوسية ( توليد درجة التعادل الحراري ) والمصاحبة للتبادل الحراري نتيجة للفرق الحراري المحدود واحتكاك السوائل ومزج السوائل .... الخ في هذا البحث تم تحليل سخانات المياه الشمسية (SDHW)باستخدام القانون الثاني للديناميكية الحرارية مع الاهتمام بشكل خاص بتحليل وتعظيم سخانات المياه الشمسية الديناميكية والحرارية وتغطى عقبات متطلبات تسخين المياه .

كما تم تمثيل (SDHW) حسابياً بإستعمال المعلومات الحقيقية للمناخ وجميع متطلبات تسخين المياه حسب راند (RAND) وفترو (VITRO) و (ASHRAE) وتم تمثيل نظام تسخين المياه بإستخدام احجام مختلفة للخزانات ومساحات الحصول على درجة التعادل الحراري المتولدة وكمية الطاقة المستهلكة .

هذا ولقد تم تمثيل كل حالة على مدى ٢٤ ساعة وذلك بتقليل درجة التعادل الحراري لنفس المدة وتم مقارنة درجة التعادل الحراري والطاقة المستهلكة من كل حالة مع النتائج المستحصل عليها من التمثيل الحسابي .

ولقد نتج عن تعظيم (SDHW) لأقل درجة تعادل حراري مولدة انخفاض في درجة التعادل الحراري المولده واستهلاك الطاقة المساعدة ، إن دراسات التعظيم لأي نظام تسخين شمسي تظهر أن درجة التعادل الحراري بكل لتر مستهلك من الماء الساخن هي ليست دالة قوية للتدفق . ونتج عن الدراسة أن أقل درجة للتعادل الحراري والطاقة المستهلكة يجب ان تساوي تقريباً حجم الخزان ليوم واحد ، وان نسبة حجم الخزان الى الجامع يجب ان تتراوح بين ٥٠ ـ ٧٠ .

# CHAPTER 1

# INTRODUCTION

## 1.1 General Introduction

The development of civilization is clearly a reflection of human struggle to discover and control the various energy resources. The development of agriculture, the invention of the steam engine, and the use of coal and oil are all important milestones in this struggle. Each step has brought a dramatic change in man's way of life. We are on the threshold of a shift from oil, gas and coal to new sources of energy, such as wind, nuclear, and solar energy. The ever depleting resources of oil and coal, coupled with a strong public opposition in installing new nuclear facilities has made it more important to investigate the potentials of clean alternative energy resources. One of these energy resources is solar energy, which is available in abundance. Also, the cost of energy from all of the non-renewable sources has risen to the point where the economics of solar energy utilization have become far more favorable than at any time in the past. Solar energy can be utilized in heating and cooling of buildings, driving irrigation pumps, generating electrical power, and in general reducing the demands on conventional energy resources.

One of the recent areas of major interest relates to the use of domestic hot water heaters for solar energy utilization. The primary objective of this investigation is the analysis and optimization of solar domestic hot water (SDHW) systems according to the second law of thermodynamics.

Solar water heating has great potential and is now recognised as a viable technology for domestic and industrial use. Years of rigorous research and development in universities, industry and government laboratories in the world have produced a large variety of practical systems for hot water supply and space heating with solar energy. Domestic water heaters, which is the topic of this research, represent the largest use for medium temperature solar collectors and the largest monetary volume for all solar products. Over 90 percent of solar water heaters sold consist of a solar collector, insulated water tank, operating "module" comprising one or more pumps, controller, valves and in many systems, a heat exchanger.

Traditionally solar systems are analyzed on the basis of the first law of thermodynamics which dictates that energy in a system is not lost, but it is transmitted from one medium to another or it is transferred from one form to another, However first law doesn't account for irreversibilities (entropy generation) accompanied during heat transfer across finite temperature difference, fluid friction, mixing of fluids, etc. The operation of any solar system is thermodynamically irreversible due to three main features: the sun - collector heat exchange, the system ambient heat loss and friction loss within system circuit. In other words the process of solar energy collection is accompanied by the generation of entropy upstream of the collector, downstream of the collector and depending on configuration inside the other components. In order to harvest solar energy efficiently and to place available work at the disposal of humanity for consumption, attention should be focussed on the minimization of entropy generation in system components and optimization of systems accordingly, without compromising the basic engineering function of the systems. An effective and concise way to accomplish this crucial task is to develop a sound thermodynamic framework of the system and analyze it on the basis of second law of thermodynamics. This aspect of energy conversion has received considerable attention in the literature. Second law analysis quantifies the collection and useful consumption of available energy and pinpoints the unrecoverable losses, leading the way to improving the thermodynamic performance system. It can be used together with energy analysis to define the optimum system which satisfies the imposed thermal and economic constraints and minimizes the available energy loss.

A comprehensive survey of literature was conducted and a summary of all related studies is presented in Chapter 2. Analytical expressions of the components used in this investigation are derived in Chapter 3. Computer simulation results are detailed in Chapter 4, which also highlights in sufficient depth, the impact of parameters such as collector area, storage volume and load on the system performance. Chapter 5 is concerned with the results and discussions of optimization studies. Conclusions and recommendations for future work are outlined in Chapter 6. Chapter 7 includes the references cited in this thesis. Appendices include Fortran program codes.

### 1.2 Objectives of this research:

The principal objective of this thesis is to analyze and optimize solar hot water systems for minimum entropy generation while satisfying dynamic, thermal, total hot water requirement constraints. Specifically the following objectives are to be fulfilled:

1. Common solar hot water system configurations, as shown in Fig.(1.1) and Fig.(1.2), will be considered and they will be simulated using actual

weather data while providing the necessary hot water requirement at constant temperature in accordance with Rand, Vitro and ASHRAE load profiles. In each case the entropy generated and auxiliary energy needed will be determined. Systems will be simulated for various tank volume over collector area and total load mass flow over tank volume ratios to determine their effect on entropy generation and auxiliary power consumption. During these simulations collector area will be kept constant and effect of variations in collector flow rate will also be investigated.

2. For every case simulated, the distribution of load mass flow during 24 hour period will be obtained by minimizing total entropy generation during the same period. For each case, the total entropy generated and the auxiliary power needed will be compared with the simulation results to see if entropy generation and the auxiliary power depend heavily on the load profile. In each optimization study, the total load mass flow will be kept equal to the corresponding simulated case.

3. A parallel objective is to study the sensitivity of system performance to variations in system parameters such as storage tank volume and total load on the system.

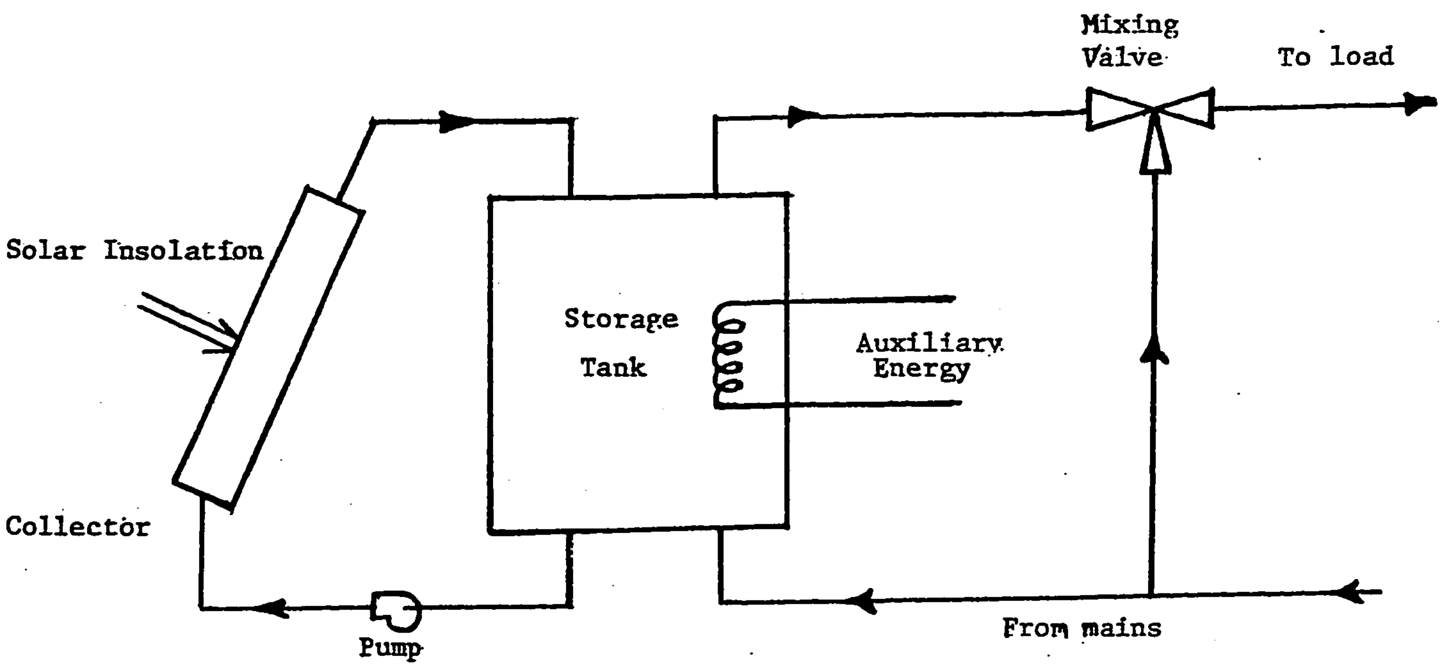



Fig. 1.1 Solar domestic hot water system#1

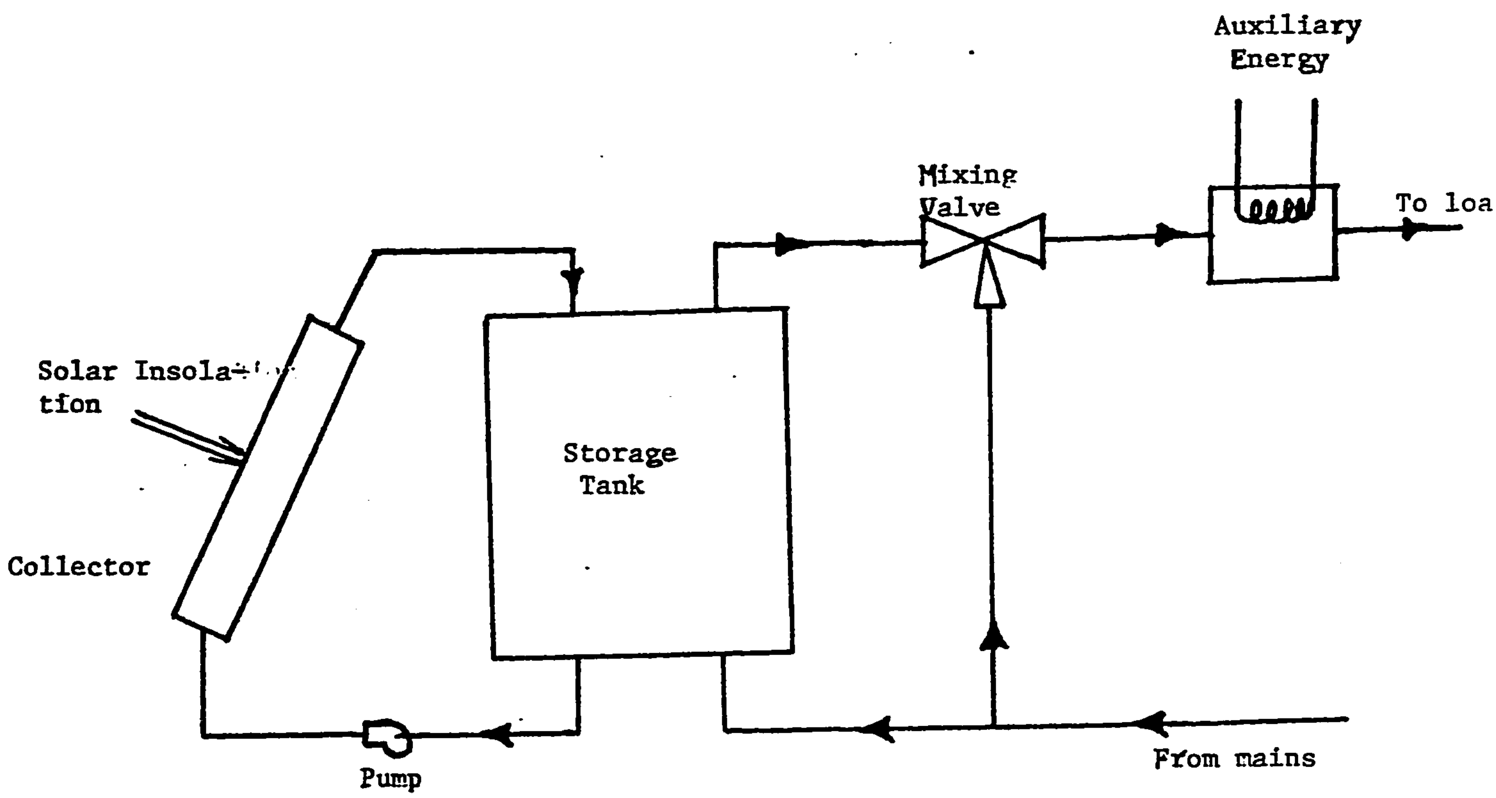



Fig. 1.2 Solar domestic hot water system#2

# CHAPTER 2

# LITERATURE SURVEY

## 2.1 Introduction

The basic ideas of the second law of thermodynamics have started initially with the observations of Carnot in 1824 which resulted in a statement of certain limitations in transforming heat into useful work using heat engine cycles. Later, Rudolf Clausius in 1850 developed one alternative statement of the second law of thermodynamics: heat by itself cannot pass from a hotter body to a colder body without causing other changes in the environment. Lord Kelvin in 1851 formulated another alternative statement of the second law: a process whose only effect is the complete conversion of heat into work cannot occur. There are other macroscopic definitions and all are valid and equivalent to each other.

The entropy change $\Delta S$ occasioned in a system by transfer to it of an amount of heat $\Delta Q$ was first introduced into thermodynamics by Clausius in 1865 as:

$$\Delta S \geq \frac{\Delta Q}{T} \tag{2.1}$$

where T is the absolute temperature of the system where $\Delta Q$ is the heat absorbed. The definition of entropy in this form allows us to restate the second law in a form that requires all natural processes to proceed in the direction of increasing entropy. This was also starting point of the ever continuing big fight in science and philosophy [1]. Since then the entropy concept has been extended into information theory [2], economics [3], biology [4], environmental planning [5], art [6], ecology [7], and other areas as well [8-9]. In engineering area, the entropy concept has led to many different applications starting with Keenan [10-14] who used the property "availability" and made it popular in USA. Today, the term "exergy" introduced by Rant [11] is also used widely [11,13].

The deepening energy crisis of 1973 renewed the curiosity in utilization of second law for more efficient analysis of thermal systems [11-14] and there has been an explosion of interest expressed by engineering and academic community in the analysis of thermal systems from the second law point of view, and several monographs and textbooks appeared in the published literature [13-20]. Hence the application of second law has enhanced significantly over the past 15 years.

Liu and Wepper's [21] classification of the second law analysis encompasses practically all the areas of thermal systems research. According to them, second law literature can be divided as follows:

1. Theory and fundamental aspects of second law analysis, teaching methods
2. Energy utilization, energy policy
3. Thermo-economics, available energy cost analysis
4. Cryogenic equipment and processes
5. Desalination processes
6. Fluid flow and heat transfer equipment and processes

7. Solar energy systems

8. Complex energy systems (electrochemical, geothermal, nuclear and hydrogen energy systems)

9 Mass transfer and separation equipment, and processes

10. Combustion and chemical reaction processes

11. Fuel conversion processes

12. Synthesis of energy-efficient chemical processes.

Of the foregoing classification, the topic of our interest is the application of second law analysis to solar energy systems, therefore the present literature survey is confined to the works related to the following:

1. Solar collectors

2. Solar collectors coupled with storage tank and auxiliary heater.

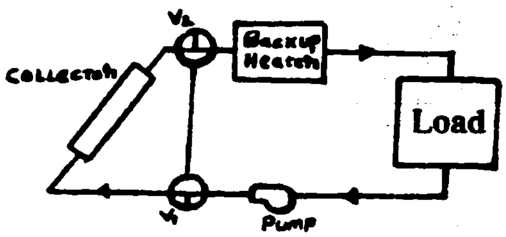
### 2.1.1 System description and operation

A solar water heating system is basically a simple device consisting of solar collectors, hot water storage tank, heat exchanger, auxiliary heater, mixing valve, pumps, piping, and controls.

Solar collector being the heart of solar system, is the pivotal research topic of many earlier investigations. It is a special kind of heat exchanger that transforms solar radiant energy into heat. Regions characterized by mild climate usually have a collector and storage tank with a fluid flow maintained by either natural convection or forced convection. In regions where the ambient temperatures touch subzero levels, a closed loop heating system employing antifreeze solutions in the collector loop and heat exchanger, has to be employed. Closed loop systems are also used to avoid scaling in the risers and consequent loss of efficiency and life. In a closed loop system, it is expected that the efficiencies are lower than those obtained in the open loop systems. Additionally the antifreeze fluids are relatively more expensive and some of these are combustible or even toxic. In every climate, one always encounters days when insolation is not sufficient to provide the required energy input. It is, therefore, common to integrate auxiliary heating element into a solar hot water system to make the system suitable for round the year use. The augmentation of the solar system with an auxiliary energy source is also desirable from the point of view of economic considerations, since the necessary collector area corresponding to minimum insolation conditions is too large to be cost-effective. For domestic and commercial applications, one has to employ a conventional system, whose capacity is large enough, to meet demand for hot water even in the absence of sunshine.

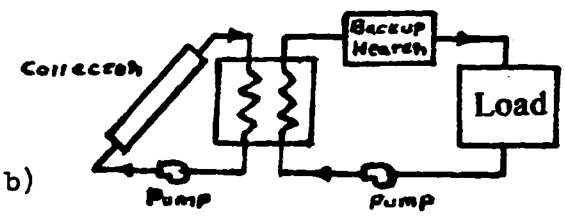
Schematic representation of typical solar domestic hot water heating systems are shown in Fig.(2.1). Operation of a typical system is essentially as follows: Fluid from water mains is passed through the collectors at a constant or variable flow rate whenever the collectors can deliver useful heat. Water enters the collector at certain inlet temperature and the outlet temperature varies with insolation. The heated water flows into the storage tank. Whenever hot water is available in the tank, it is withdrawn to the process at constant flow rate. The water in the tank should be well mixed because that ensures best storage utilization [22].

(a)



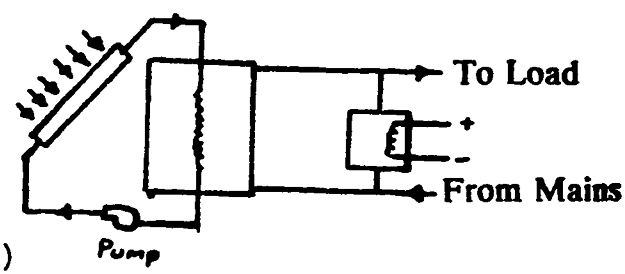

Flow diagram for process heat system without storage and without heat exchanger. [25]

(b)



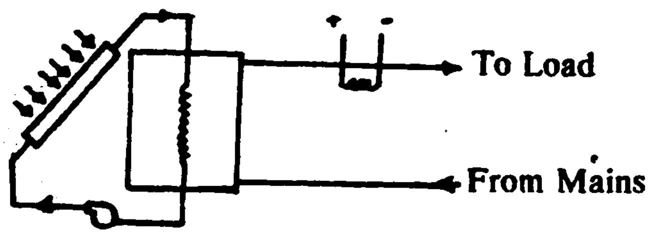

Flow diagram for process heat system without storage: with heat exchanger. [25]

(c)



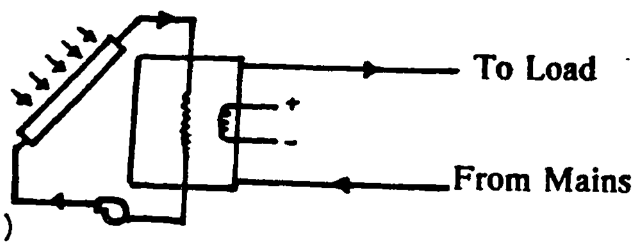

Solar heating system with two storage devices, one solar and the other electrical backup. [33]

(d)



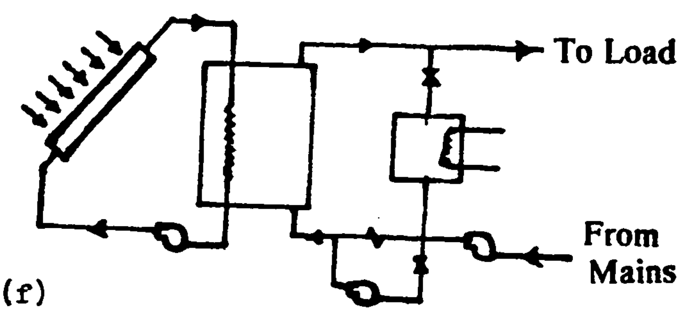

Solar heating system with electrical backup heating on demand. [33]

(e)




Solar heating system with electrical backup heating in storage. [33]

(f)




Solar heating system with two storage devices, one solar and the other either electrical backup or solar. [33]

Fig. 2.1

Schematic representation of typical solar domestic hot water heating systems.

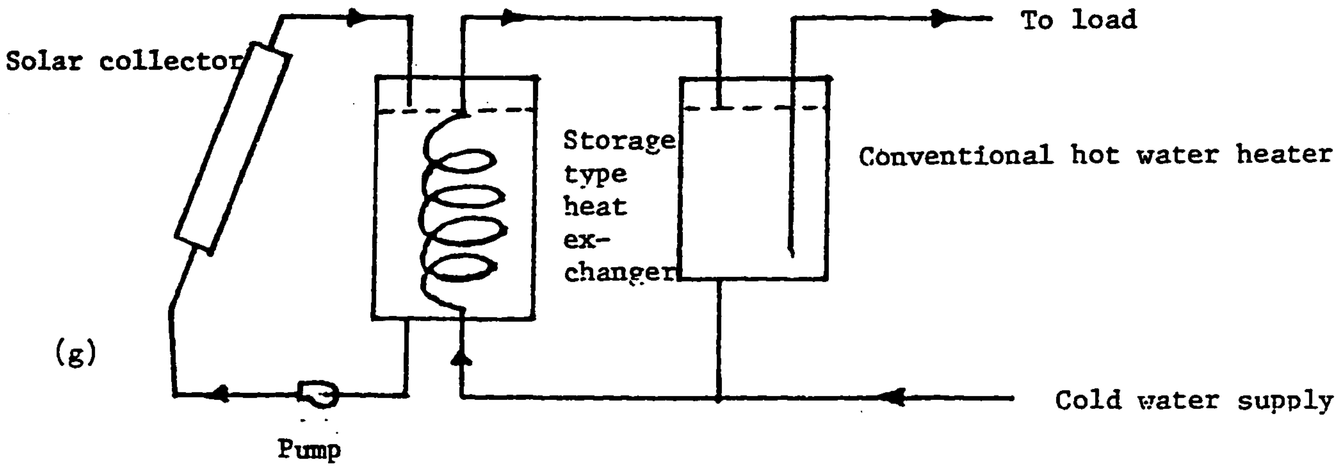



Fig. 2.1 Schematic of storage type heat exchanger.
Solar Domestic hot water system.[19]

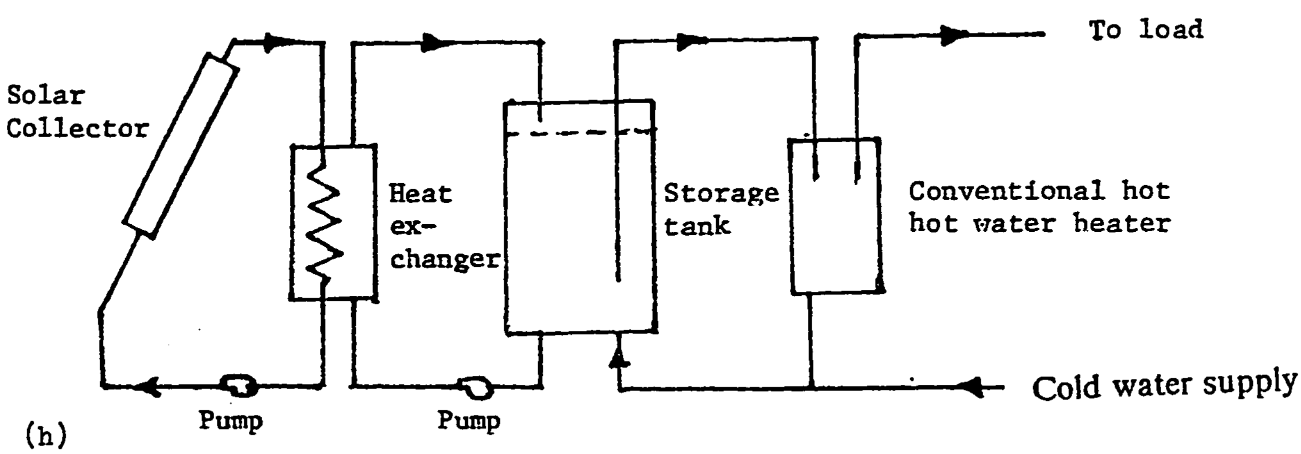



Fig. 2.1 Schematic of heat exchanger - storage tank.
Solar domestic hot water system.[19]

The backup heater may be immersed in the storage tank as in Figure (2.1e) or it may be in series with the storage system as in Fig.(2.1d). It operates at variable heat rate to bring the fluid up to the desired temperature. Provision of auxiliary energy in the storage tank results in higher collector mean temperature, poorer collector performance and higher requirement on auxiliary energy [26]. On the other hand providing auxiliary energy in series with the storage tank. maximizes the use of the solar collector output.

The system is fitted with tempering or mixing valve that mixes cold water from mains with heated water from the storage tank to put an upper limit on the temperature of the hot water going to the load.

## 2.2 First Law Analysis of Solar Systems

A great deal of work has been done in this area. Lof and Karaki [27] carried out an extensive study on the performance of various solar hot water heating systems (both used at research centres and at fields). They compared the first law performance of several systems and arrived at a conclusion that the performance of systems in well controlled test buildings is considerably higher than the best system in the field. It has been suggested that the performance could be dramatically improved by enhancing the soundness of design, quality of installation and maintenance.

Fisher and Fanney [28] compared the performance of SDHW systems subjected to Rand, Vitro and ASHRAE hourly load profiles (Figures 2.2 - 2.4) having the same total daily load. Study reveals that a system supplying hot water in accordance with the Rand profile slightly outperforms the system subjected to Vitro profile. It is inferred that the most favorable time to withdraw

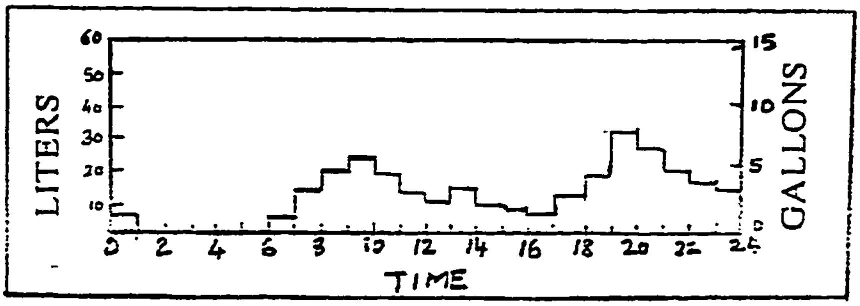



Fig.(2.2) Rand Hourly Load Profile

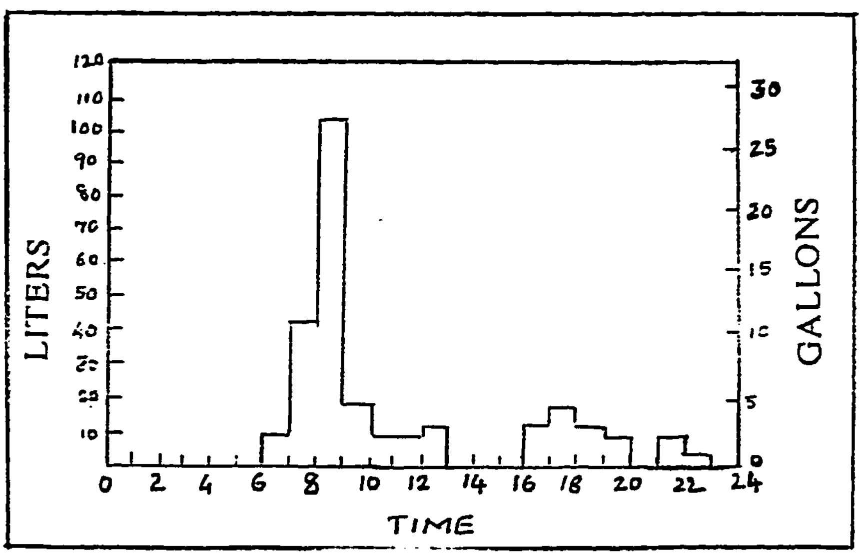



Fig.(2.3) Vitro Hourly Load Profile

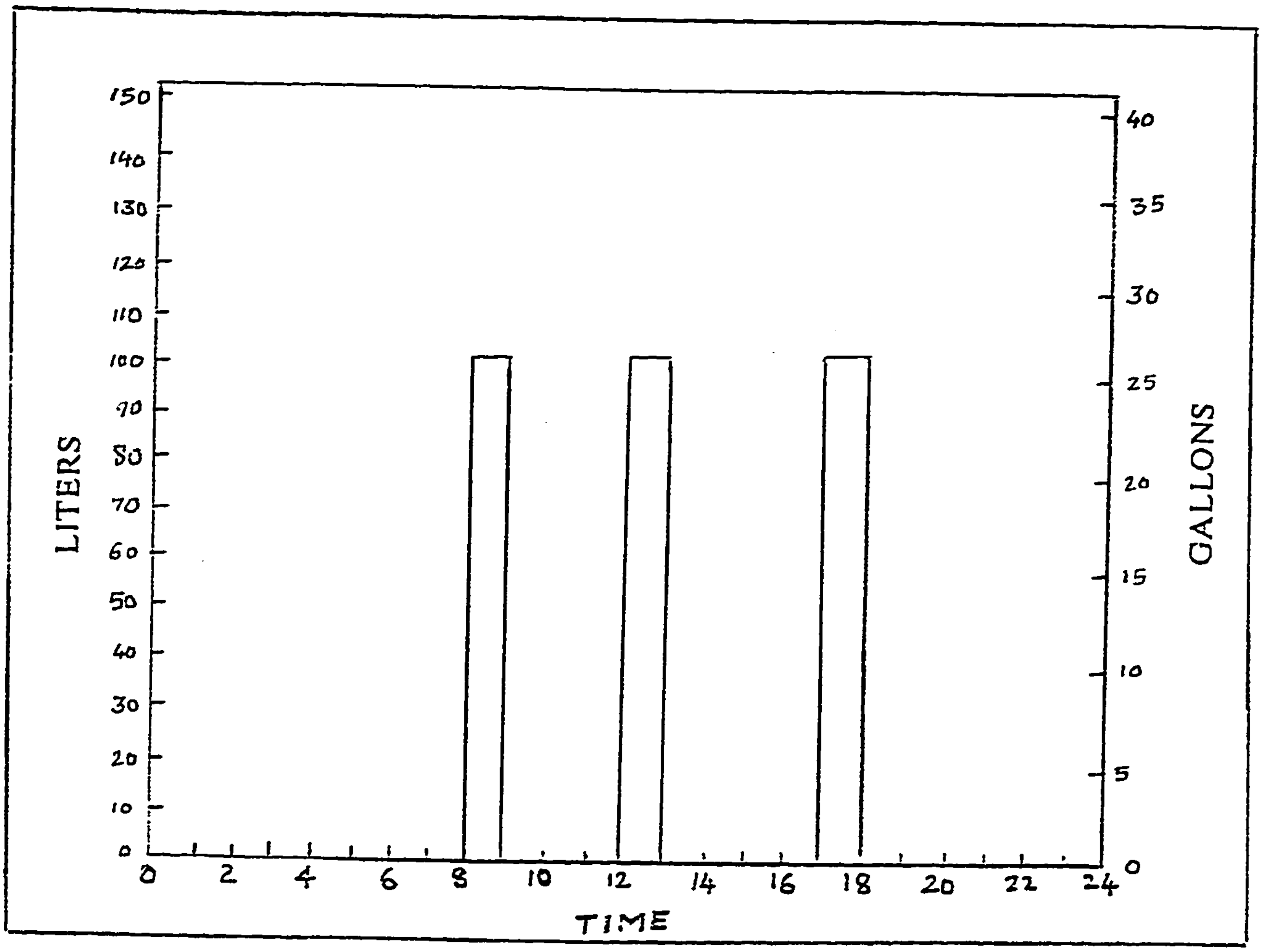



Fig.(2.4) ASHRAE Hourly Load Profile

water from a SDHW system is during collection of solar energy and the least favorable time to withdraw a large percentage of the daily hot water load is early morning.

Zollner [29] estimated the monthly performance of solar domestic hot water heating systems subjected to Rand hourly load profile and concluded that evening and morning weighted draw patterns effect the performance by 20 percent.

Newell and Althof [30] have presented a generalized design methodology for deriving an analytical, simplified solar system performance prediction model. Lumped model analysis has been utilized to yield particular solutions. The method offers an efficient and inexpensive means for examining the feasibility of various concepts related to system design, such as storage tank temperature variations. Constant hourly load profile has been used in the study.

### 2.3 First Law Control of Solar Systems

The rate at which energy is collected by a solar collector can be increased by increasing the flow rate through the collector. However, increasing the flow rate increases the power required to push the fluid through the collector circuit. Dynamics of the system also affects the solar energy collection process, therefore an optimization is required to control flow rate by performing a trade-off between the energy collected and the power required.

With the increasing importance being attached to applications of solar energy, considerable amount of work has been devoted to the control of such systems. The controller of solar energy system is quite similar to the controller of any dynamic system that is subject to disturbances. The disturbances in solar system refer to static or dynamic constraints due to changing weather conditions, energy requirements and dynamic behavior of the system components. The primary function of the controller is to control pump settings for the collection and distribution of solar energy, secondary functions involve the control of pump and valve settings for freeze protection, high temperature protection etc. Most active solar hot water systems are controlled by conventional on/off controllers which are simple, inexpensive and reliable. With its high flow rate and correspondingly low temperature rise across the collector, the on/off system operates at a high energy collection efficiency. Excessive pump cycling caused by on/off controllers may be overcome by proportional control in which the fluid flow rate in the collector loop is varied in proportion to the temperature difference between the collector and storage. These two types of controllers are compared and it was found that, although the proportional control requires considerably more parasitic pumping energy, under variable insolation, and it is more complex to implement, its thermal energy gain is higher than on/off controllers [31]. Winn [32] has presented the characteristics of bang-bang (on/off), proportional, optimal controllers etc., and their applications to solar-energy systems.

Numerous investigators have carried out extensive study on the optimization of flow rates through solar collectors, and have stressed that Modern Optimal Control theory promises to be a very appropriate tool for the design of solar collector systems under constraints. A performance measure representative and indicative of the system behavior can be defined and Pontryagin's Maximum Principle can be used to determine the optimal control law.

Significant amount of works have examined solar collector and solar system from the energy point of view and performed several optimizations on the performance of these systems. Various performance measures have been proposed in the literature and implemented in practice. Adaptive and optimal control techniques are used to control HVAC systems in large solar heated and cooled buildings by employing an integral quadratic cost functional of the form :

$$J=1/2\int_0^T (x^TQx+u^TRu)dt \tag{2.2}$$

where x is the deviation from the point of operation of the plant state vector representing such variables as the load temperature and storage tank temperature, u is the deviation from the operation point of the input vector representing the control variables such as flow rates, and Q and R are weighting matrices which assign relative importance to the various state and control variables [33].

One of the studies maximizes the increase in storage tank temperature as the performance measure [34], others minimize the total auxiliary energy used to meet the given demand by using a performance index of the form

$$J=\int_0^T f.Q_{aux}.dt \tag{2.3}$$

where f represents the utility rate structure and $Q_{aux}$ is the auxiliary power needed [35-37]. A quadratic performance index of the form

$$J=\int_0^T \{f.Q^2_{aux}+C.(T_e-T_{set})^2\}dt \tag{2.4}$$

is selected for minimization for a complex solar energy system, where f represents the utility rate structure, C is a comfort weighting coefficient, $T_{set}$ is the desired temperature and $Q_{aux}$ is the auxiliary power used [37-38]. The first term in Eq.(2.4) represents consumption of auxiliary energy and the second term represents a penalty for deviation from the desired enclosure temperature. This type of performance index which includes an energy utilization term and a discomfort term is very common in the literature.

One of the first applications of Pontryagin's Maximum Principle to solar systems considers a performance measure of the form [39].

$$J = \int_0^T (Q_c - P)dt \tag{2.5}$$

where $Q_c$ is the energy collected by the collector and P is the power loss for pumping the fluid through the collector. Although only a single state variable is used, namely the storage temperature, an explicit solution could not be obtained due to the nonlinearities in $Q_c$. The resulting two point boundary value problem is solved numerically in that study. The solution obtained could not be implemented in a practical controller because it was not a function of measurable variables of the system.

A later study [40] presents an approximate analytical solution to the problem which shows close agreement to the numerical solution obtained earlier. Using the same performance index (Eq. 2.5), the optimal collector flow rate is obtained for several system models with and without the power loss term, and it is found that maximization of the energy collected without power loss results in a bang-bang control. However, the optimal control is not bang-bang when the power loss term is included, regardless of the constraints due to collector and storage dynamics, and an explicit equation for the collector mass flow rate is obtained as :

$$m_{opt} = \left[\frac{C_1 A_c^{\,2} U_L (F')^2 [S - U_L(T_{in} - T_a)]}{2\alpha C_2 C_p}\right]^{1/(\alpha+1)} \tag{2.6}$$

where

| | |
|---|---|
| $m_{opt}$ | Optimum mass flow rate |
| $A_c$ | Collector area |
| $U_L$ | Overall loss coefficient |
| $F'$ | Collector efficiency factor |
| $S$ | Absorbed solar energy per unit area |
| $C_p$ | Specific heat of fluid |
| $C_2$ | Constant related to friction and pipe dimensions |
| $T_{in}$ | Collector inlet temperature |
| $T_a$ | Ambient temperature |

$C_1$ is the weighting factor which takes into account the difference between the cost of energy to run the fluid through the collector circuit and that to provide heating, $C_1$ and $\alpha$ set arbitrarily by experience.

The performance measure in Eq.(2.5) is used to find the optimum flow rate with a mix of different collector brands for a recirculating solar energy system in which energy storage is utilized; and for a once-through system where the working fluid does not return to the collector [41]. Another study uses Klein's Plug-Flow Model to describe the dynamics of a liquid flat-plate collector to obtain the maximum net useful energy gain in Eq.(2.5) over each day by controlling the fluid flow rate in the collector [42].

Chato [43] obtained an optimal hourly load profile for solar batch water heaters by optimizing the total energy gain. A major conclusion reached is that load profile can significantly effect the system performance. It has also been concluded that for optimum energy gain, the hot water usage should be shifted toward later afternoon and away from early morning hours.

The performance measure in Eq.(2.5) compares the value of mechanical energy (which is converted into heat through fluid friction) and the thermal energy gain of the collector. The required pumping power is reduced to thermal energy in the collector and in this process the value of mechanical energy is not lost due to the first law of thermodynamics, but the quality of it is degraded. In other words, the work producing potential (available energy or exergy) of mechanical energy is destroyed because of friction.

The second law of thermodynamics in such situations provides a scientific basis for controlling the solar collecter systems where the exergy potential is degraded due to the thermodynamic irreversibilities, such as heat transfer across a finite temperature difference, mechanical friction, mixing of dissimilar fluids.

## 2.4 Second Law Analysis of Solar Systems:

Second law analysis aims at identifying and eliminating the causes of thermodynamic irreversibilities. It is a powerful technique for achieving the highest degree of thermodynamic efficiency. And since the maximization of thermodynamic efficiency is one of the objectives of fundamental research in thermal systems, this methodology deserves to occupy a central place in all the compartments of thermal systems research. Unlike traditional thermodynamic analyses, this method invokes the second law and focusses for the first time on the minimization of thermodynamic irreversibility.

Although the objective of second law analysis is always the same - the identification and minimization of thermodynamic irreversibilities (entropy generation) without compromising the basic engineering function of the system - the analysis can be conducted either in terms of entropy generation or in terms of lost available work. These two alternatives are recommended by Gouy Stodola's theorem (Eq. 2.7), which states that the lost available work is directly proportional to the entropy production, the proportionality factor is the absolute environment temperature $T_o$

$$W_{lost} = T_o S_{gen} \geq 0 \tag{2.7}$$

**2.4.1 Entropy Analysis:**

In another work [25], open-loop and closed-loop solar heating systems were simulated using hourly meteorological data. The predicted temperature variations were used to obtain the yearly entropy generation and the mean second law efficiency. For open-loop systems, the second law efficiency has a maximum value for an optimal collector area, when the collector plate temperature is used as the source temperature. When the equivalent 'sun' temperature is employed, the second-law efficiency decreases steadily with collector area. Similar behaviours are observed for closed-loop systems. In general, higher load temperature and more efficient collectors produce higher second-law efficiencies for both systems.

**2.4.2 Exergy (Availability) Analysis:**

As a general thermodynamic concept, exergy is the maximum work achievable from a system via a hypothetical reversible process, as the system reaches a state of equilibrium with the environment. In real processes, the exergy potential is destroyed, at least partially, whenever thermodynamic irreversibilities (heat transfer across a finite temperature difference, mechanical friction, mixing etc) are present. It is important to emphasize that available energy (exergy) analysis is a useful method to complement, not to replace, energy analysis. They together define the optimum system which satisfies the imposed thermal and economic constraints and minimizes the available energy loss.

There are many expressions derived in the literature for second law efficiency (based on availability or exergy of radiation) of solar powered devices, and profound controversy exists on determination of the right expression. An expression of the form :

$$\eta_{max} = 1\text{-}f(\frac{T_o}{T_s}) \tag{2.8}$$

is suggested by many researchers where

where

$T_o$ is the ambient temperature.

$T_s$ is the equivalent black body temperature of the sun.

Every researcher derived his own $f(\frac{T_o}{T_s})$ function.

Gribik and Osterle[44] tried to resolve the controversies over the proper expression for the theoritical maximum conversion efficiency of a solar device. According to them the second law efficiency attainable by a solar powered device is given as :

$$\eta_{max} = 1-(\frac{4T_o}{3T_s}) \tag{2.9}$$

In view of the high exergy content of solar thermal radiation, an important engineering task is to find ways to maximize the collection of exergy from the insolation stream. This task is analogous to finding means to avoid the irreversible destruction of exergy in the process of collection and delivery to a potential user.

### 2.5 Second Law Control of Solar Systems:

#### 2.5.1 Control Through Entropy Minimization:

Bejan [45] considered a simple constant temperature model for operation of a solar collector. By analyzing the operation of the collector on the of basis second law, he demonstrated that the irreversibility or entropy production in solar collectors is attributed to heat transfer processes occuring between sun and collector, between collector and ambient and inside the collector circuit. Optimum operating conditions for minimum heat transfer irreversibility are derived. However, this work does not take into account the power loss and the mass flow rate becomes infinite in some working regions.

#### 2.5.2 Control Through Exergy Maximization:

The performance of solar collectors has been examined from the stand point of exergy and the optimum flow regime is determined [46]. Attention was mainly concentrated on the fundamental thermal design problem of maximising the exergy extraction from solar collectors with energy storage capability under time varying conditions. The important inference reached in the analysis is that in a collector installation with thermal inertia, the optimum temperature for maximum exergy delivery must vary in step with the insolation and that the collector operation at constant temperature brings 4 to 8% drop in exergy output relative to a theoritical maximum.

Fujiwara [47] evaluated the performance of flat plate solar collectors from the stand point of exergy by defining exergy efficiency as the ratio of exergy gain of the collector to the solar insolation on the collector. This efficiency is expressed as

$$\eta_e = \frac{\Delta\varepsilon}{IA_c} \qquad \Delta\varepsilon = \Delta h - T_a \Delta s$$

$$= \left(\frac{mC_p}{IA_c}\right)\left\{T_o - T_i - T_a \ln\left(\frac{T_o}{T_i}\right)\right\} \tag{2.10}$$

where

| | |
|---|---|
| $\Delta\varepsilon$ | Exergy gain in the collector |
| m | Mass flow rate |
| $C_p$ | Specific heat of collector fluid |
| I | Solar radiation incident on the collector |
| $T_o$ | Collector outlet temperature |
| $T_a$ | Ambient temperature |
| $T_i$ | Collector inlet temperature |

Eq. (2.10) results in infinite flow rate for a certain range of inlet temperature. This is unsuitable for optimum condition because infinite pumping power is required to realise this condition. This difficulty was resolved by introducing pressure drop in Eq.(2.10), as it causes a decrease in exergy. This prevents the flow rate from becoming infinite because pressure drop (exergy destruction) will increase with flow rate. It is suggested from this work that the mean fluid temperature ought to be kept constant for optimum control.

Chelghoum and Bejan [48] performed quantitative analytical and numerical study on the exergy delivery potential of solar collectors with energy storage capability. They showed that the daily regime of collector operation (filling & draining) has a profound effect on the exergy delivery capability and this regime can be selected by design to maximize the delivery of solar exergy to the user. They also concluded that the total exergy delivered by the installation is maximum when the temperature is maintained at an optimum constant level throughout the insolation period.

Kar [49] suggested that the optimum operation of flat-plate solar collectors can be investigated on the basis of output exergy efficiency, defined as the ratio of exergy output $E_o$ of a collector to the maximum exergy output $E_{o,max}$ (corresponding optimum inlet temperature) under the same solar input and ambient temperature as :

$$\eta_e = \frac{E_o}{E_{o,max}} \tag{2.11}$$

Kar [50] performed an exergy optimization on flat- plate collectors with no collector and storage dynamics. By utilizing the optimal control strategy, an expression for optimum mass flow rate was obtained which is given as :

$$m_{opt} = \left[\frac{A_c^{\,2} U_L (F')^2 [S - U_L(T_{in} - T_a)](1 - \frac{T_a}{T_{in}})}{6C_f C_p}\right]^{1/4} \tag{2.12}$$

where

| | |
|---|---|
| $m_{opt}$ | Optimum mass flow rate |
| $A_c$ | Collector area |
| $U_L$ | Overall loss coefficient |
| $F'$ | Collector efficiency factor |
| $S$ | Absorbed solar energy per unit area |
| $C_p$ | Specific heat of fluid |
| $C_f$ | Coefficient related to friction and pipe dimensions |
| $T_{in}$ | Collector inlet temperature |
| $T_a$ | Ambient temperature |

A comparison was made between energy optimization and exergy optimization (i.e between Eq.(2.6) and Eq.(2.12)) by setting $C_1 = (1 - \frac{T_a}{T_{in}})$, $\alpha = 3$, $C_2 = C_f$ in Eq.(2.6). It is important to notice that first law optimization leaves $C_1$ arbitrary, but the second law optimization provides a value in terms of system variables. It was found that the optimum mass flow rate obtained by exergy optimization provides 1.4% less energy, but 25% more exergy and reduces the power loss by 92% for 25°C inlet temperature.

The optimum mass flow rate given by Eq.(2.12) was used in output exergy efficiency, Eq.(2.11) and an expression for optimum output exergy efficiency was obtained in the following form [51]:

$$\eta_{opt} = \left\{1 - \frac{2A_c F' U_L}{3 m_{opt} C_p}\right\} \frac{[S - U_L(T_{in} - T_a)](1 - \frac{T_a}{T_{in}})}{[S - U_L(T_{opt} - T_a)](1 - \frac{T_a}{T_{opt}})} \tag{2.13}$$

where $T_{opt}$ is the optimum collector inlet temperature. Other parameters are as defined above.

A recent study [52] discusses in detail the use of exergy analysis, rather than energy analysis, for the evaluation of thermal energy storage systems. It is reported that the use of exergy analysis clearly takes into account the loss of availability of heat in storage operations and, hence it reflects more correctly the thermodynamic and economic value of the storage operation.

# CHAPTER 3

# DEVELOPMENT OF MODELS

## 3.1 Introduction

In this chapter, focus is on the development of an analytical performance model of the components of the solar hot water heating systems used in this investigation as shown in Figures (3.1-3.2), with special emphasis on the second law aspects. The rational design of a solar thermal system requires a knowledge of the dynamic interaction of all the system components. Theoretical models are tools required to assist in predicting the thermal performance of systems and are also helpful in ensuring the proper sizing of the solar equipment.

Analytical models based on transient thermal performance of all the components involved are possible to obtain. Such models are usually presented in the form of simultaneous non-linear differential equations. Analytical solutions of these models are difficult to obtain, however numerical solutions are possible. The magnitude of the time constants of such systems are in the order of minutes. Generally meteorological data are recorded at hourly intervals. As these intervals are larger than the time constants, this data cannot be used to predict the transient performance of the system within hourly intervals.

Simplified models based on steady state energy equations could be used to obtain solutions for the systems from the available data. Due to the nature of the problem at hand, the steady state energy model for collector and transient model for storage tank are considered to be satisfactory for predicting the per-

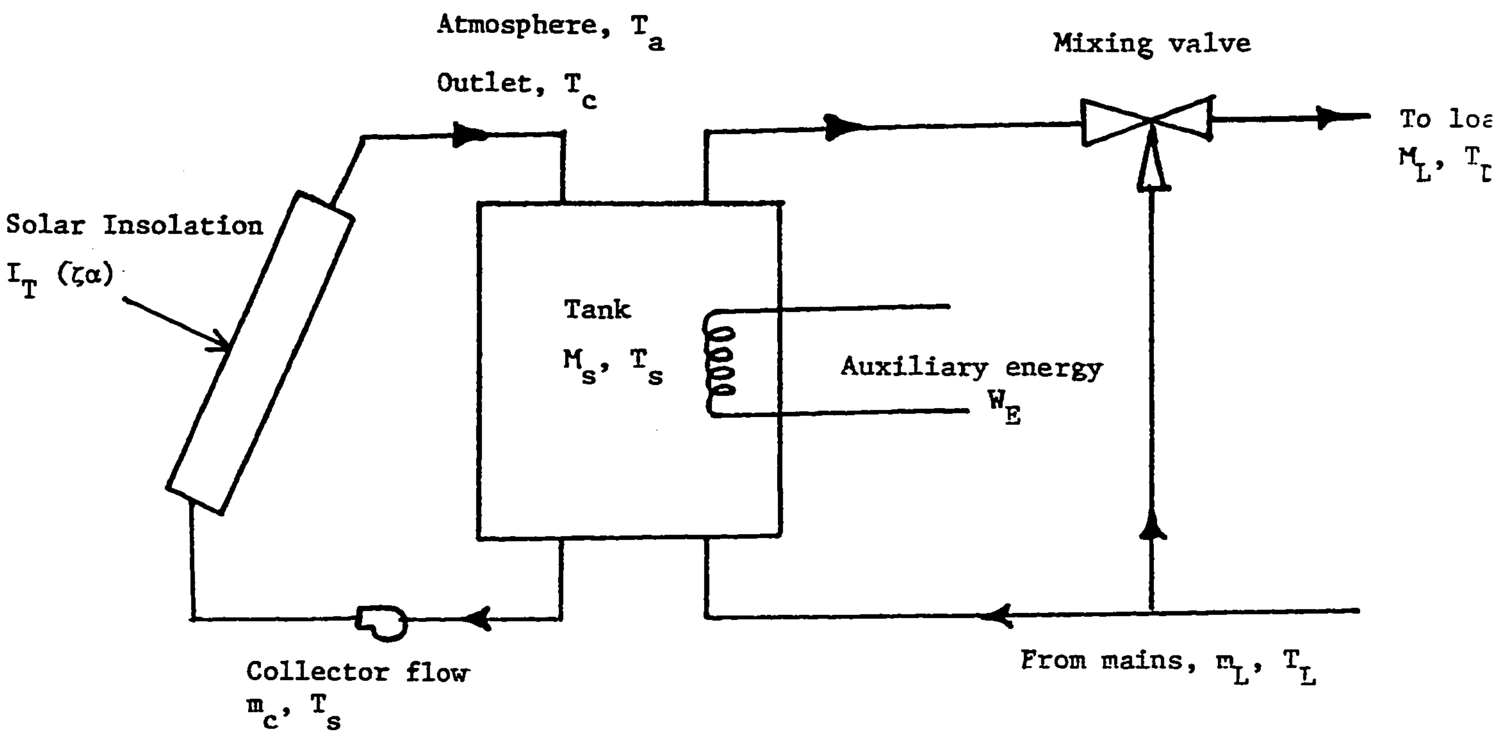



Fig. 3.1 Solar domestic hot water system#1

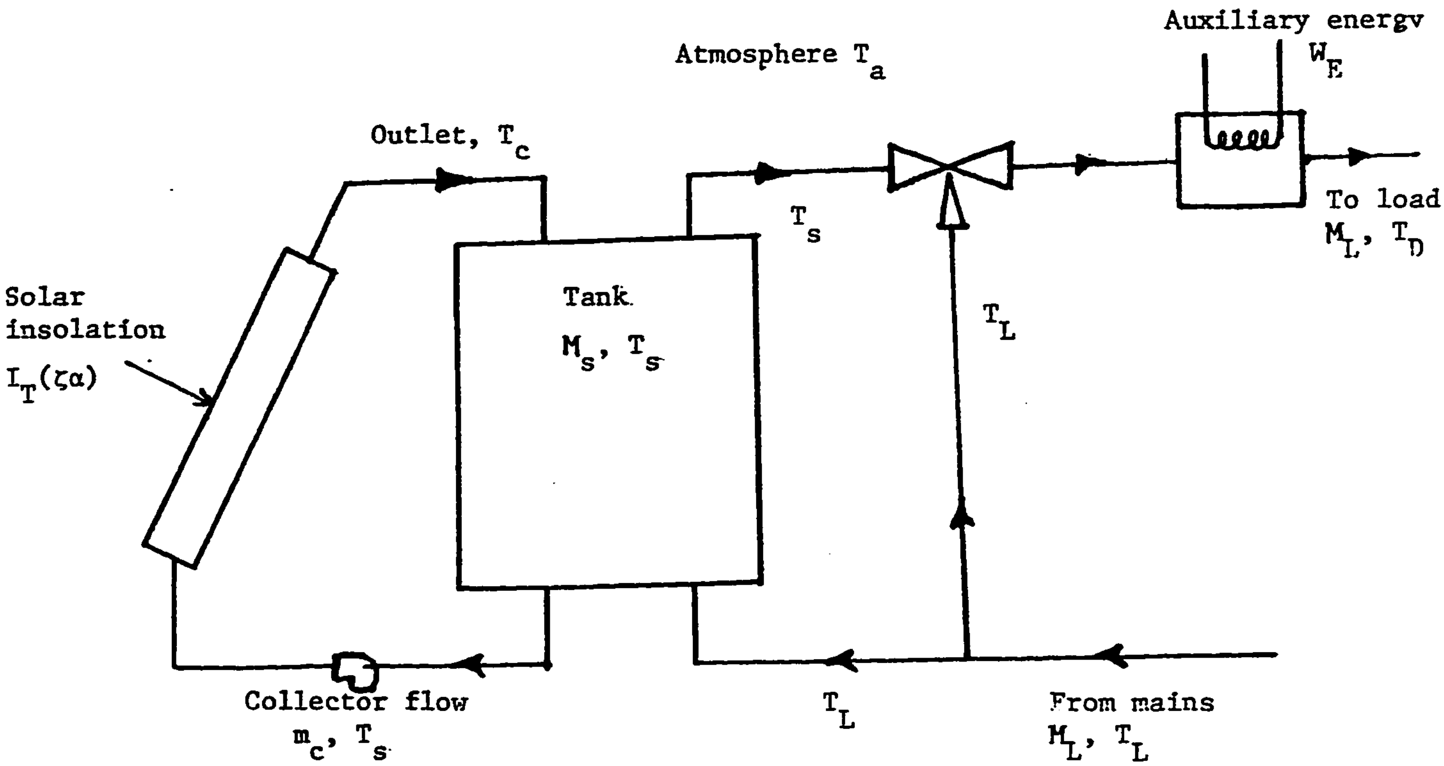



Fig. 3.2 Solar domestic hot water system#2

formance of the system. The following sections entail the discussion of the concept of control volume and entropy generation, the development of models and the associated entropy generation equations.

### 3.2 Concept of Control Volume, and Entropy Transfer:

The analysis of an open thermodynamic system (control volume) requires the selection of an appropriate boundary drawn around the region of interest. As shown in Fig.(3.3) the control surface can have one or more openings (ports) for mass transfer. The open system analysis consists of making three statements regarding the three system properties: mass(M), energy(E) and entropy(S). The first is a mass conservation statement:

$$\sum_{in} m - \sum_{out} m = \frac{\partial M}{\partial t} \tag{3.1}$$

Mass transfer = Mass change in the control volume

For a steady open system $\frac{\partial M}{\partial t} = 0$, therefore Eq.(3.1) becomes:

$$\sum_{in} m - \sum_{out} m = 0 \tag{3.2}$$

The first law of thermodynamics dictates:

$$\sum_{in} m(h + \frac{1}{2}v^2 + gz) - \sum_{out} m(h + \frac{1}{2}v^2 + gz) + Q - W_{sh} = \frac{\partial E}{\partial t} \tag{3.3}$$

Energy transfer = Energy change in the control volume

If the operation is steady and if the specific kinetic and gravitational energies of specific streams $m_{in}$, $m_{out}$ are insignificant, eventually Eq.(3.3) takes the form

$$\sum_{in} mh - \sum_{out} mh + Q - W_{sh} = 0 \tag{3.4}$$

Second law of thermodynamics states:

$$\sum_{in} ms - \sum_{out} ms + \frac{Q}{T} \leq \frac{\partial S}{\partial t} \tag{3.5}$$

Entropy transfer ≤ Entropy change in the control volume

where the specific entropy 's' is representative of the thermodynamic state of each stream right at the system boundary. For the purpose of evaluating the degree irreversibility of actual engineering systems, it is convenient to rearrange the second law inequality (Eq. 3.5) in an elegant form as:

$$S_{gen} = \frac{\partial S}{\partial t} + \sum_{out} ms - \sum_{in} ms - \frac{Q}{T} \geq 0 \tag{3.6}$$

This form defines the rate of entropy generation in the system as, a quantity always positive and in the reversible limit equal to zero. It makes sense to use $S_{gen}$ as a measure of a system's departure from reversibility. $S_{gen}$ solely depends on the degree of thermodynamic irreversibility of the system. The task of improving the thermodynamic performance of an engineering system requires the proper identification of those features (components) most guilty of entropy production. If the engineering systems and their components are to operate such that the destruction of available work is minimized, then the design of such systems and components must begin with the minimization of entropy generation.

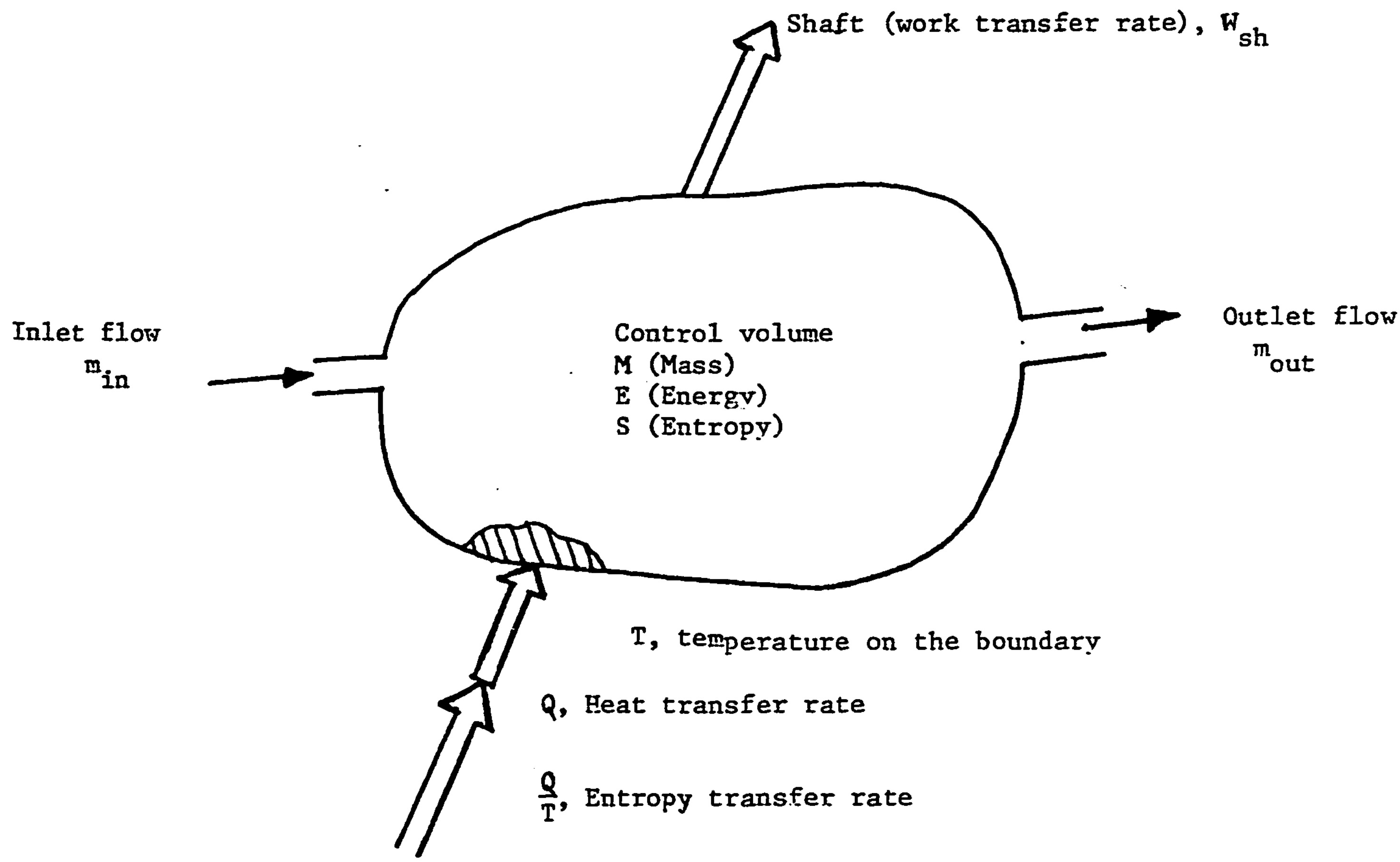



Fig. 3.3 Control volume and its interactions with environment.

If control volume is in steady state, then $\frac{\partial S}{\partial t}$ term vanishes.

### 3.3 Collector Model and Entropy Equation:

Several different simplified models to examine the thermal performance of collectors exist in literature. The zero capacitance model of Hottel, Whillier, and Bliss (HWB) is widely accepted for accurately depicting collector performance and is recommended [26] to be the best suited for simulation because of its computational simplicity and its excellent agreement with more sophisticated models. The HWB model expresses the rate of energy collection $Q_u$ as

$$Q_u = A_c F_R \left[ I_T(\tau\alpha) - U_L(T_s - T_a) \right] = m_c C_p (T_c - T_s) \tag{3.7}$$

where

| | |
|---|---|
| $A_c$ | Collector area ($m^2$) |
| $I_T$ | Rate at which solar radiation is incident on the collector surface per unit area ($W/m^2$) |
| $(\tau\alpha)$ | Effective transmittance absorptance product |
| $T_s$ | Temperature of the fluid entering the collector K |
| $T_c$ | Collector outlet temperature K |
| $T_a$ | Outside ambient temperature K |
| $m_c$ | Collector flow rate (kg/sec) |
| $C_p$ | Specific heat of the collector working fluid (J/Kg K) |
| $U_L$ | Collector overall energy loss coefficient ($W/m^2$ K) (kept constant in simulation) |

The collector heat removal factor $F_R$ is defined as:

$$F_R = \frac{m_c C_p}{A_c U_L}\left[1 - \exp\left(\frac{-A_c U_L F'}{m_c C_p}\right)\right] \tag{3.8}$$

where $F'$ is the collector efficiency factor.

The second law of thermodynamics in steady state form, Eq.(3.6), can now be employed to determine the entropy generation in the collector. In accordance with the sign convention of Figure (3.3) and neglecting the irreversibility associated with friction in the collector flow passage, entropy generation of the collector can be written in the following form [18,25] :

$$S_c = \frac{-I_T(\tau\alpha)A_c}{T_p} + \frac{A_c U_L(T_s - T_a)}{T_a} + m_c C_p \ln\left(\frac{T_c}{T_s}\right) \tag{3.9}$$

where $T_p$ is the plate temperature

$$T_p = T_s + \frac{Q_u/A_c(1 - F_R)}{U_L F_R} \tag{3.10}$$

$$T_c = T_s + \frac{A_c F_R}{m_c C_p}\left[I_T(\tau\alpha) - U_L(T_s - T_a)\right] \tag{3.11}$$

The first term of Eq.(3.9) is the rate of entropy decrease in the input energy source at temperature $T_p$. The second term is the rate of increase of entropy in the surroundings due to the heat loss from the collector control volume. The last term represents the rate of entropy increase in the collector fluid, from inlet to outlet.

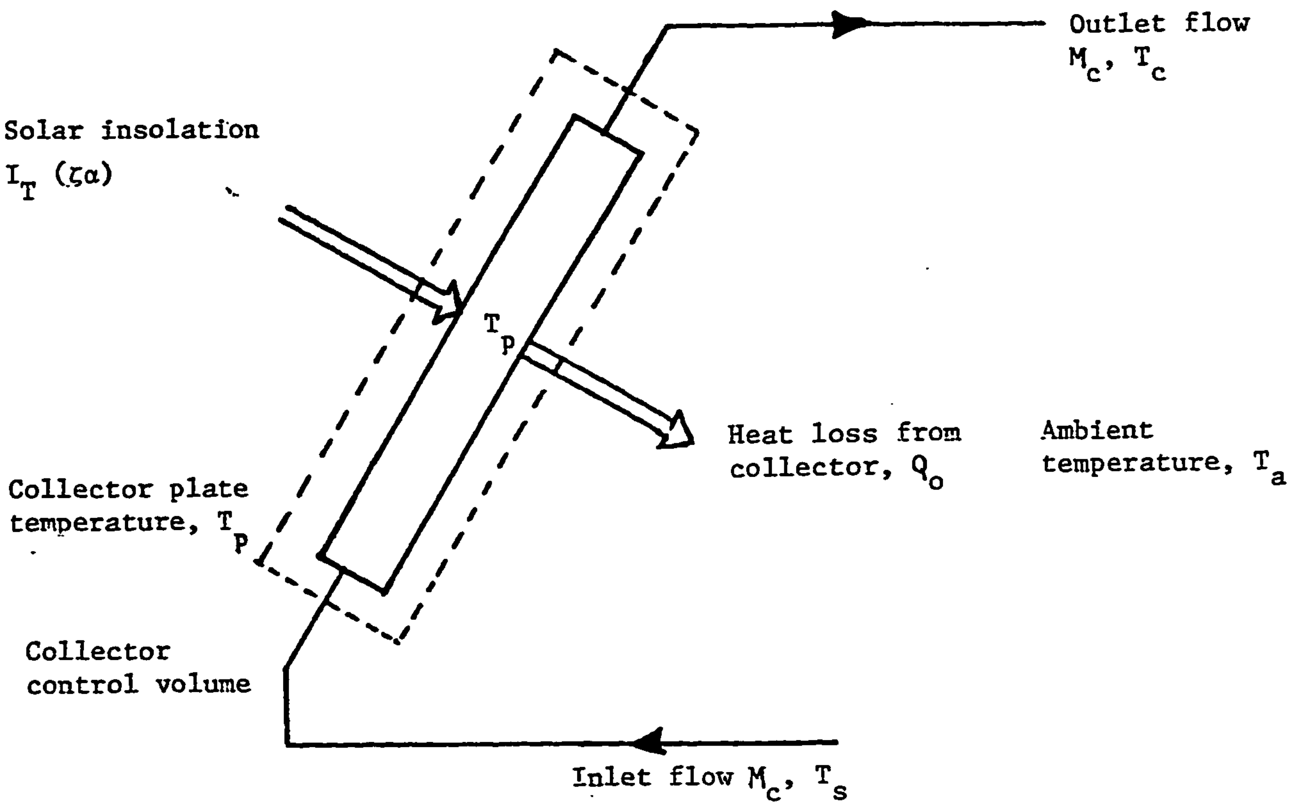



Fig. 3.4 Schematic of Solar Collector

### 3.4 Storage Tank Model and Entropy Equation:

Storage is essential for efficient collector utilization and thus to make a large contribution to our total energy consumption. In many solar energy systems energy storage is in the form of sensible heat of water. Water storage units may operate with significant degrees of stratification, that is with water not at uniform temperature over the vertical dimensions of the tank. As a conservative estimate, a simple model having uniform temperature over the vertical dimensions has been used to represent the storage tank.

The energy storage capacity of a water (or other liquid) storage unit at uniform temperature operating over a finite temperature difference

$$Q_s = (mC_p)_s \Delta T_s \tag{3.12}$$

Where $Q_s$ is the total heat capacity for a cycle operating through the temperature range $\Delta T_s$, with 'm' kilograms of water in the unit. The temperature range over which such a unit operates is limited at the low extreme because of the nature of the processes and at the upper limit due to the vapor pressure of the liquid.

For a well mixed tank as shown in Fig.(3.5), an energy balance on the tank yields:

$$M_s C_s \frac{dT_s}{dt} = Q_u - Q_L - Q_{loss} \tag{3.13}$$

In this equation $Q_u$ is the rate of addition of energy from the collector given by Eq.(3.7), $Q_L$ is the rate of removal of energy from the tank to load given by :

$$Q_L = m_c C_p (T_s - T_l) \tag{3.14}$$

and $Q_{loss}$ is the external heat loss of the tank given by :

$$Q_{loss} = U_s A_s (T_s - T_a) \tag{3.15}$$

where

$U_s$ Heat loss coefficient of the tank

$A_s$ Surface area of the storage tank

Implicit in the discussion is the assumption that irreversibility associated with friction is small. Application of the second-law of thermodynamics to sensible heat storage system yields the entropy generation equation as follows [18-25]:

$$S_s = \frac{d}{dt}\left[M_s C_s \ln(\frac{T_s}{T_0})\right] + \frac{U_s A_s (T_s - T_a)}{T_a} + m_c C_p \ln(\frac{T_s}{T_c})$$

$$+ m_L C_p \ln(\frac{T_s}{T_L}) \tag{3.16}$$

The first term in Eq.(3.16) is the rate of increase of entropy in the tank, second term is the entropy increase in the surroundings, the third term represents the change in entropy of the collector fluid and last term represents the increase in entropy of the load fluid. $T_o$ is the reference temperature.

$$\frac{d}{dt}\left[M_s C_s \ln(\frac{T_s}{T_0})\right] = M_s C_s \frac{1}{T_a} \frac{dT_s}{dt} \tag{3.17}$$

Substituting for $\frac{dT_s}{dt}$ from Eq.(3.13), we obtain the entropy generation in the

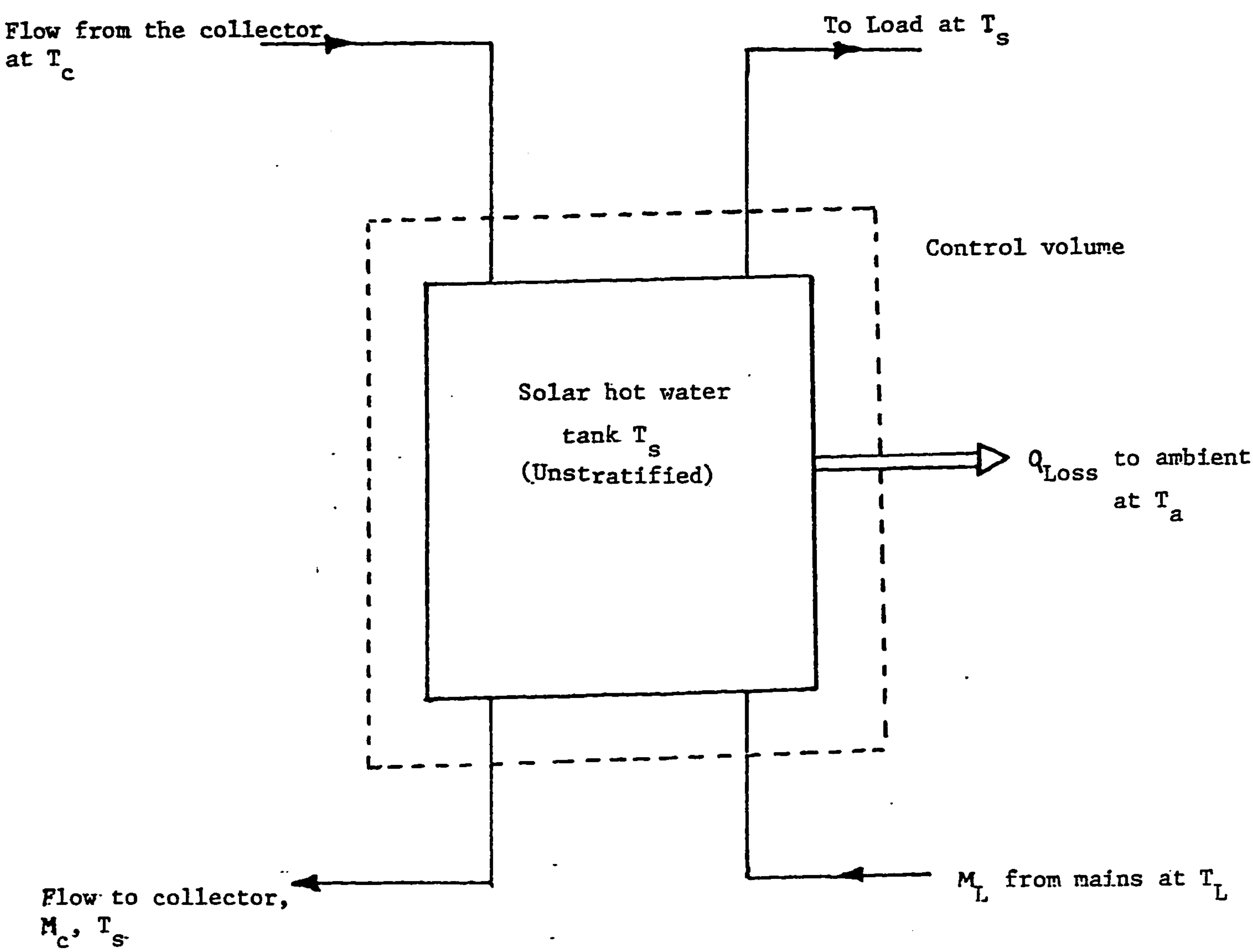



Fig. 3.5 A Well Mixed Water Storage Tank

tank as:

$$S_s = \frac{A_cF_R}{T_s}\left[Q_s - U_L(T_s - T_a)\right] - \frac{U_sA_s(T_s - T_a)}{T_s} - \frac{m_LC_p(T_s - T_a)}{T_s} + \frac{U_sA_s(T_s - T_a)}{T_a}$$

$$+ m_cC_p\ln(\frac{T_s}{T_c}) + m_LC_p\ln(\frac{T_s}{T_L}) \qquad (3.18)$$

**3.5 Auxiliary Energy Model and Entropy Equation:**

The degree of reliability desired of a solar process to meet a particular load can be provided by a combination of properly sized collector and storage units and an auxiliary energy source. In a few unique areas where there seldom are clouds of significant duration, it may be practicable to provide all of the loads with the solar systems. However, in most climates auxiliary energy is needed to provide high reliability of the solar system. Auxiliary energy is useful in boosting the storage tank fluid temperature to the desired temperature $T_D$. Auxiliary energy is assumed to be pure work, therefore it is included in the tank energy balance for system shown in Figure 3.1 and the tank energy balance becomes :

$$M_sC_s\frac{dT_s}{dt} = Q_u - Q_L - Q_{loss} + W_E \qquad (3.19)$$

Since the auxiliary energy is assumed to be pure work, it does not carry with itself any entropy to the system and it is not included in entropy generation equation, Eq.(3.16). But it appears in the final form through the tank energy balance (through the $\frac{dT_s}{dt}$ term) and the tank entropy generation equation for system in Fig.(3.1) becomes :

$$S_s = \frac{A_c F_R}{T_s}\left[Q_s - U_L(T_s - T_a)\right] - \frac{U_s A_s(T_s - T_a)}{T_s} - \frac{m_L C_p(T_s - T_a)}{T_s} + \frac{U_s A_s(T_s - T_a)}{T_a}$$

$$+ m_c C_p \ln(\frac{T_s}{T_c}) + m_L C_p \ln(\frac{T_s}{T_L}) + \frac{W_E}{T_s} \qquad (3.20)$$

The entropy generation due to the auxiliary energy in system shown in Fig.(3.2) is obtained as:

$$S_{aux} = m_L C_p \ln(T_D/T_s) \qquad (3.21)$$

where

$T_D$ desired temperature

$T_s$ tank temperature

$m_L$ load flow rate

$C_p$ specific heat of the load fluid

### 3.6 Mixing Valve Model and Entropy Equation:

Mixing valve (tempering valve) is used to reduce the delivery temperature when the tank temperature is higher than the desired load temperature $T_D$. Reduction of temperature $T_s$ to $T_D$ is achieved by mixing hot water from storage tank with cold water from mains at temperature $T_L$. Mixing valve is modeled as an adiabatic and constant pressure process. Irreversibility due to friction is assumed to be small. Referring to Fig.(3.6), the continuity and steady flow energy equations can be written as :

$$m_1 + m_2 = m_L \tag{3.22}$$

$$m_1T_s + m_2T_L = m_LT_D \tag{3.23}$$

The entropy generation during the steady mixing process, using Eq.(3.6) and sign convention of Fig.(3.3), can be expressed explicitly as:

$$S_{mix} = m_Ls_L - m_1s_1 - m_2s_2 \tag{3.24}$$

where

| | |
|---|---|
| $m_1$ | Storage tank flow rate |
| $m_2$ | Cold water flow rate |

Inserting Eq.(3.22) in Eq.(3.24)

$$S_{mix} = m_L(s_L - s_1) + m_2(s_L - s_2) \tag{3.25}$$

Substitution of $m_2$ from Eq.(3.23) into (3.25), and further simplication yields:

$$S_{mix} = m_1C_p\ln(\frac{T_D}{T_s}) + (m_L - m_1)C_p\ln(\frac{T_D}{T_L}) \tag{3.26}$$

Where $m_1 = \dfrac{m_L(T_D - T_L)}{(T_s - T_L)}$

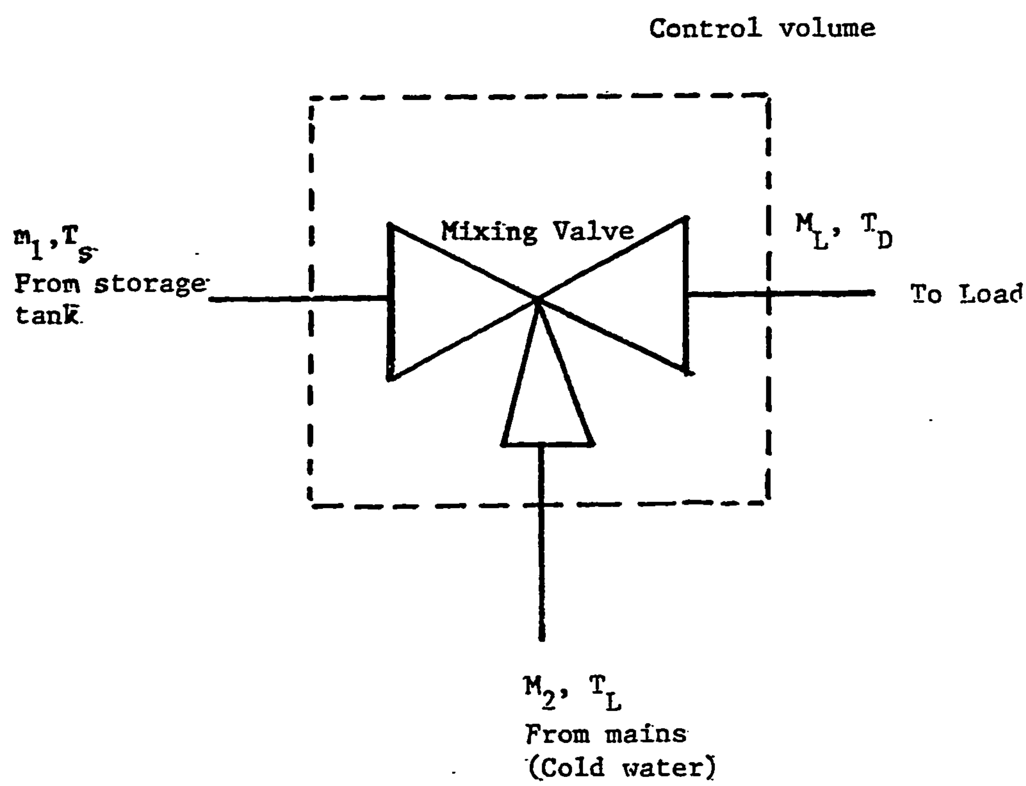



Fig. 3.6 Schematic of a Mixing Valve

# CHAPTER 4

# SIMULATION STUDIES

## 4.1 Introduction

Two solar domestic hot water systems schematically illustrated in Figures (3.1-3.2) are simulated for a 24-hour period using three different hourly load profiles: Rand, ASHRAE and Vitro, as shown in Figures (2.2-2.4). Solar insolation and ambient temperature data used in this study are for Dhahran area, dated March 1, 1985 and are depicted in Fig.(4.1). In each case, initial temperature of the tank and and the desired load temperature are taken to be 60°C, and water mains temperature is considered as 20°C. Collector area is kept constant at $5m^2$. Details of collector and storage tank parameters are furnished in Table (4.1). Variation of entropy generation and auxiliary heat requirement with storage volume and load mass flow are determined for a 24-hour period for all the three load profiles. Three different tank volumes with volume to collector area ($V_s/A_c$) ratios of 50, 70, 90 are considered. The effect of total load mass flow on the systems is investigated for each case by using total mass flows of 50, 100 and 150% of tank volume. The total entropy generation and total auxiliary energy required are calculated and compared for each case and each profile,

## 4.2 Simulation of system #1

This section explains the simulation study of the heating system #1 shown schematically in Fig.(3.1) subject to three hourly load profiles (Rand, ASHRAE

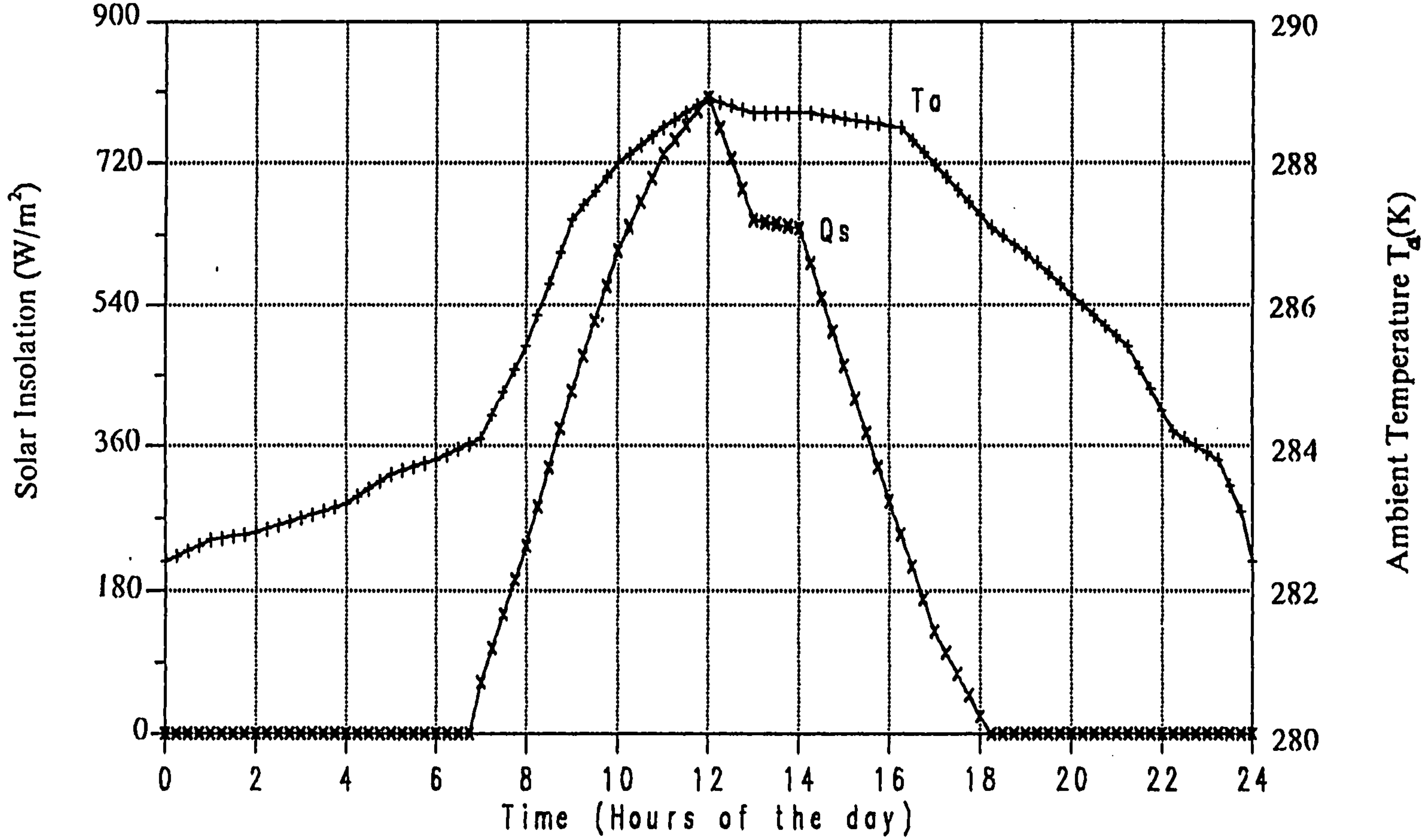



Fig. 4.1 Solar Insolation and Ambient Temperature Variation of Dhahran (March 1, 1985)

**Table (4.1) Collector and Storage Data**

| | | |
|---|---|---|
| Collector type | : | Liquid Flat Plate |
| Total collector area | : | $5 m^2$ |
| Collector flow rate | : | $.02\ Kg/sec/m^2$ |
| Collector constants | : | $F_R(\tau\alpha) = .79$<br>$F_R U_L = 3.53$<br>$F' = 0.8$ |
| Storage type | : | Water |
| Storage capacity | : | 250, 350, 450 Liters |
| Collector loss coefficient $U_L$ | : | $4.0\ W/m^2 {}^\circ K$ (kept constant) |
| Storage tank loss coefficient $U_s$ | : | $0.5\ W/m^2 {}^\circ K$ |
| Storage tank area $A_s$ | : | $6.0(V_s/1000)^{2/3}$ |
| Desired hot water temperature | : | 60 °C |

& Vitro) having the same total daily load. In this system, auxiliary heat is supplied directly into the storage tank to elevate the storage tank temperature $T_s$, if there is hot water request and if $T_s$ is less than the desired load temperature $T_D$ which is always kept at 60°C. When the temperature of heated water from storage tank is greater than the desired load temperature $T_D$ then the hot water from tank is mixed with cold water from the mains in the mixing valve to bring down the delivery temperature to 60°C.

The governing equations for the system #1 are presented below:

Dynamic equation of the tank from Eq.(3.19):

$$M_s C_s \frac{dT_s}{dt} = A_c F_R \left[ Q_{sol} - U_L (T_s - T_a) \right] - Q_{Loss} - Q_{Load} + W_E \qquad (4.1)$$

where

$Q_{Loss} = (UA)_s (T_s - T_a)$ Heat lost to the ambient

$Q_{Load} = m_L C_p (T_s - T_L)$ Load on the system

$M_s$ Mass of the water in the storage tank (Kg)

$C_p$ Water specific heat (J/kg K)

$W_E$ Auxiliary heat (W)

Entropy equations from Eqns.(3.9, 3.20 and 3.24) are given as:

**Entropy equation for Collector:**

$$S_c = \frac{-I_T(\tau\alpha)A_c}{T_p} + \frac{A_cU_L(T_s - T_a)}{T_a} + m_cC_p\ln(\frac{T_c}{T_s})$$

**Entropy equation for Storage tank:**

$$S_s = \frac{A_cF_R}{T_s}\left[Q_s - U_L(T_s - T_a)\right] - \frac{U_sA_s(T_s - T_a)}{T_s} - \frac{m_LC_p(T_s - T_a)}{T_s} + \frac{U_sA_s(T_s - T_a)}{T_a}$$

$$+ m_cC_p\ln(\frac{T_s}{T_c}) + m_LC_p\ln(\frac{T_s}{T_L}) + \frac{W_E}{T_s}$$

**Entropy equation for Mixing valve:**

$$S_{mix} = m_1C_p\ln(\frac{T_D}{T_s}) + (m_L - m_1)C_p\ln(\frac{T_D}{T_L})$$

$$S_{tot} = S_c + S_s + S_{mix} \tag{4.2}$$

where $S_{mix}$ will be zero if there is no mixing.

Simulation of the system #1 is accomplished by solving dynamic equation of the tank together with the entropy equations for a 24-hour period using a FORTRAN computer program, as given in Appendix A. Auxiliary energy is calculated from the supplied amount to the tank to bring it to the desired load temperature when load flow is needed.

Euler's integration scheme is utilized to solve Eq.(4.1). Considerable emphasis is placed on the selection of the time step to insure that the solution has reasonable accuracy and stability. Time step used in simulation is 15 minutes.

Several runs of simulation program are made with different profiles, different total load mass flow and tank volumes. Variation of tank temperature, entropy generation and auxiliary energy consumption during a day using Rand profile are shown graphically in Figures (4.2-4.4) and Figures (4.5-4.13), respectively. Similar plots are presented in Figures (4.14-4.19) for ASHRAE profile and in Figures (4.20-4.25) for Vitro profile.

As the total load flow or tank volume increases, tank temperature decreases for all the profiles, as expected. Decrease is less in Vitro profile and the tank temperature is higher than the other two profiles (since there is no load in evening hours in Vitro profile).

Variation of entropy generation and auxiliary heat with total load flow or tank volume during the hours of the day, indicates that entropy generation and auxiliary energy requirement are appreciably high during early morning hours due to the large hot water demand and less solar insolation. This finding is in good agreement with earlier works [28]. Hourly entropy generation and auxiliary energy required are higher for ASHRAE and Vitro profiles as compared to Rand profile.

Comparison of total entropy and total auxiliary energy required for Rand, ASHRAE and Vitro profiles for different tank volumes and total load mass flow rates, as listed in Table (4.2), leads to the following conclusions:

1. For a given tank size and total load mass flow, entropy generation and auxiliary energy requirement are higher in ASHRAE and Vitro profiles as compared to Rand profile. Highest values are observed in Vitro profiles. It seems that withdrawing a large percentage of hot water in early morning hours in

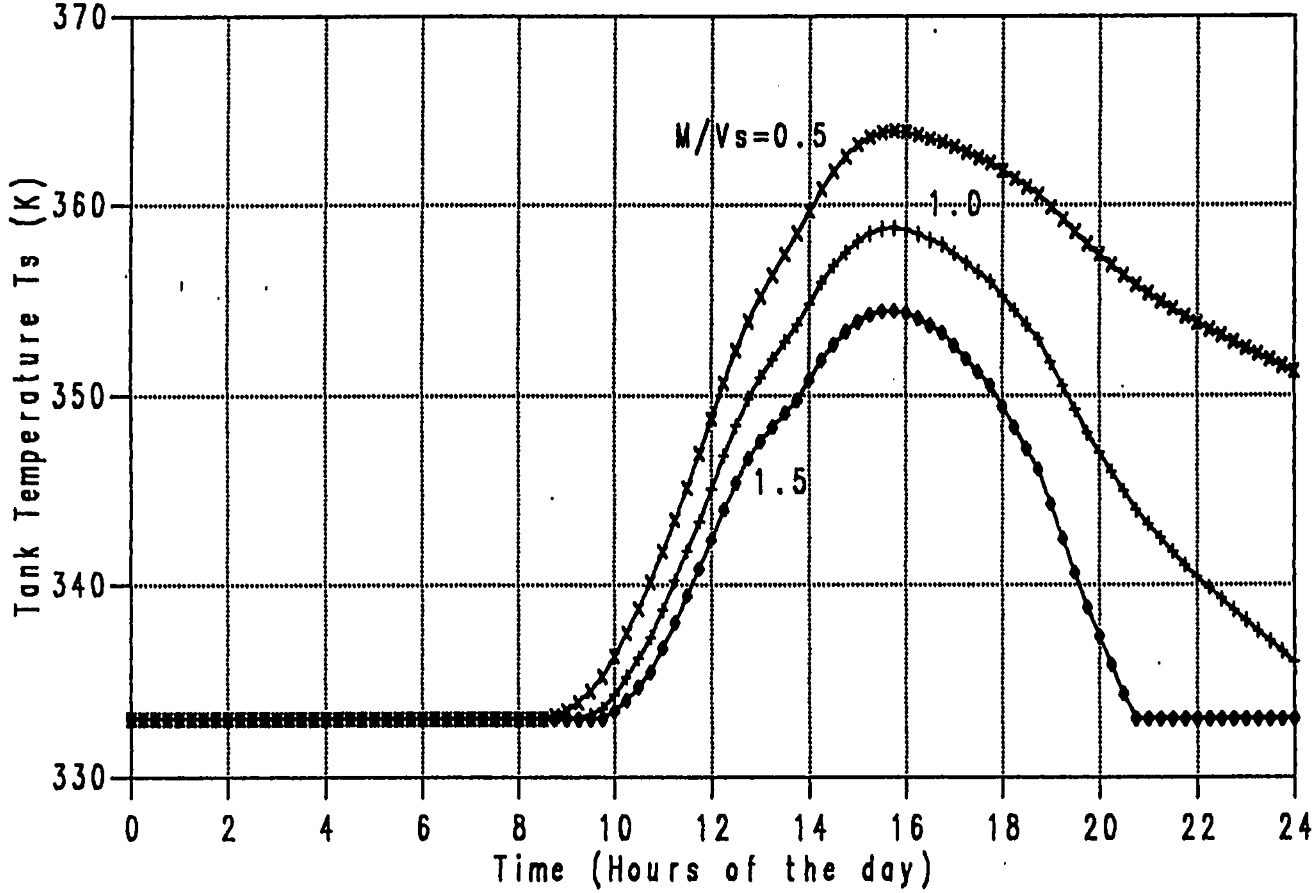



Fig. 4.2 Variation of Tank Temperature with Load
(Vs/Ac=50, Rand Profile)

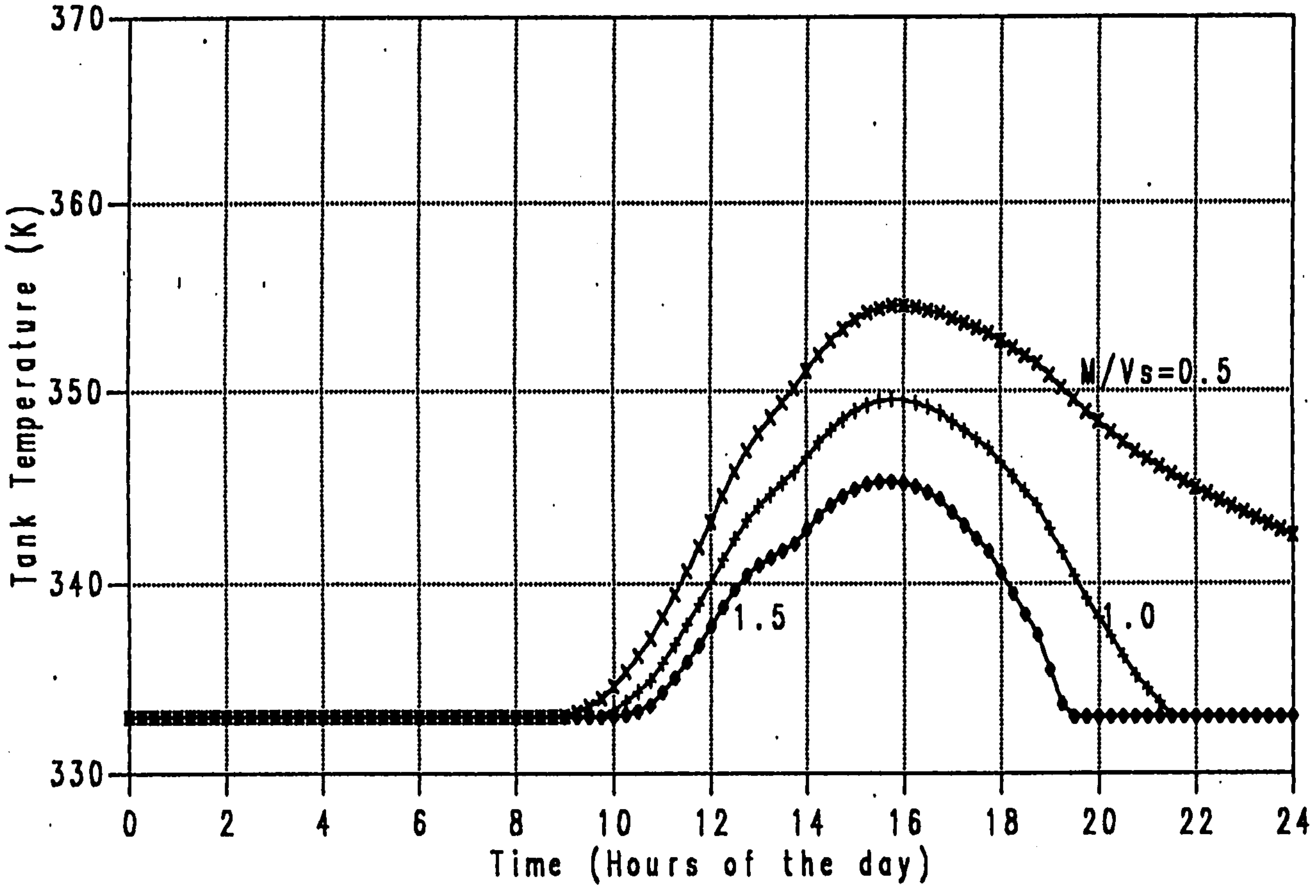



Fig. 4.3 Variation of Tank Temperature with Load
(Vs/Ac=70, Rand Profile)

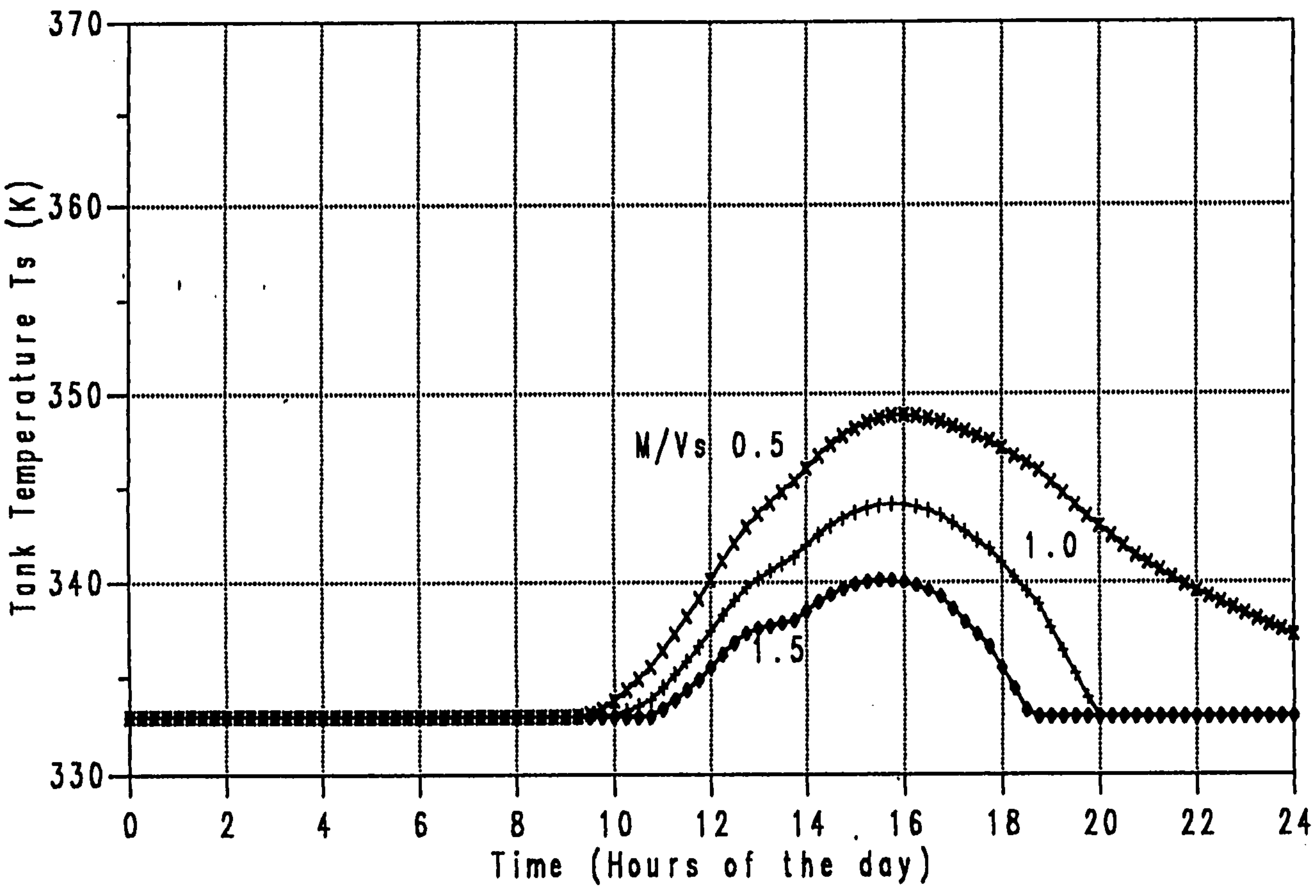



Fig. 4.4 Variation of Tank Temperature with Load
(Vs/Ac=90, Rand Profile)

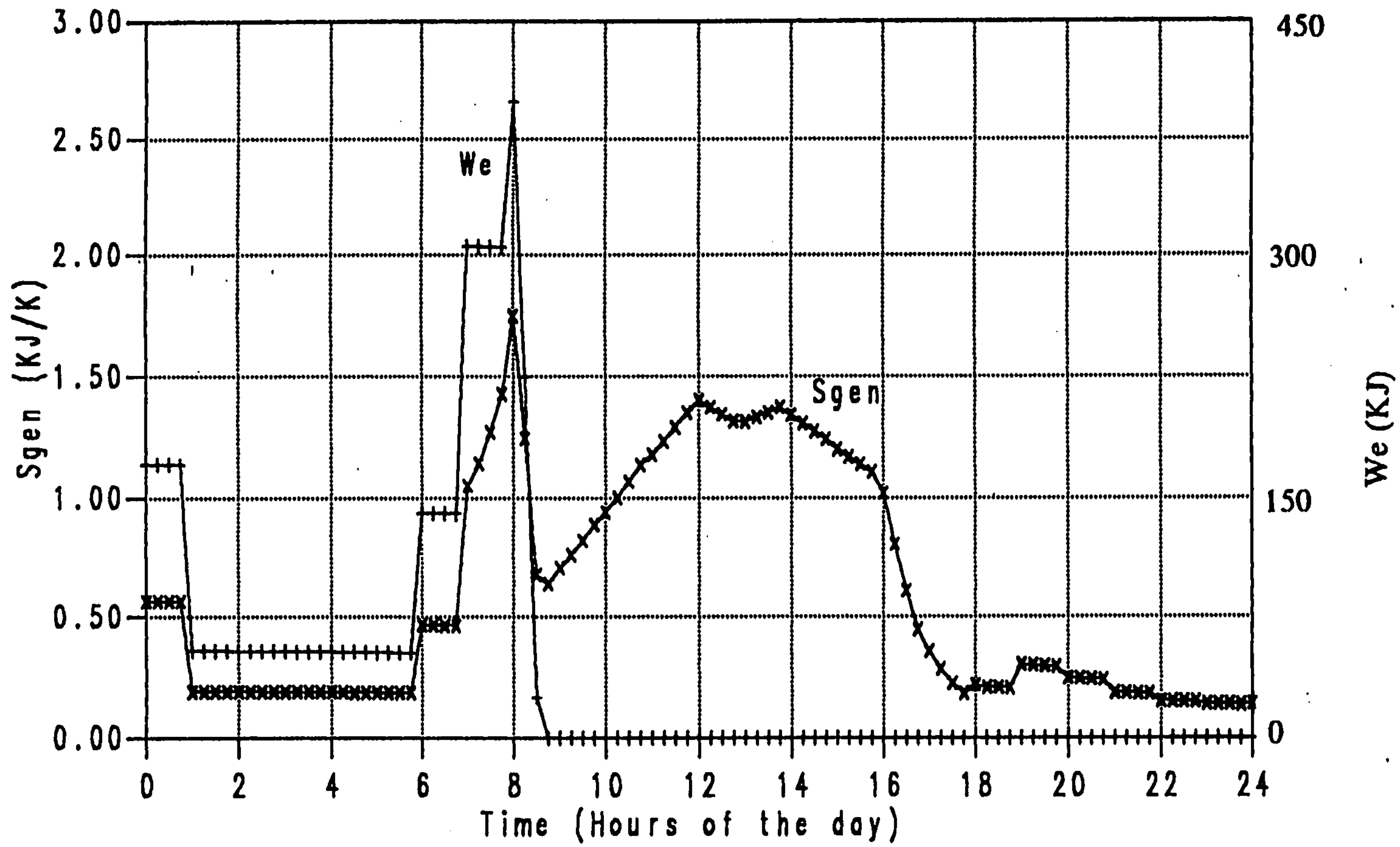



Fig. 4.5 Variation of Entropy and Auxiliary Energy (Vs/Ac=50, M/Vs=0.5, Rand Profile)

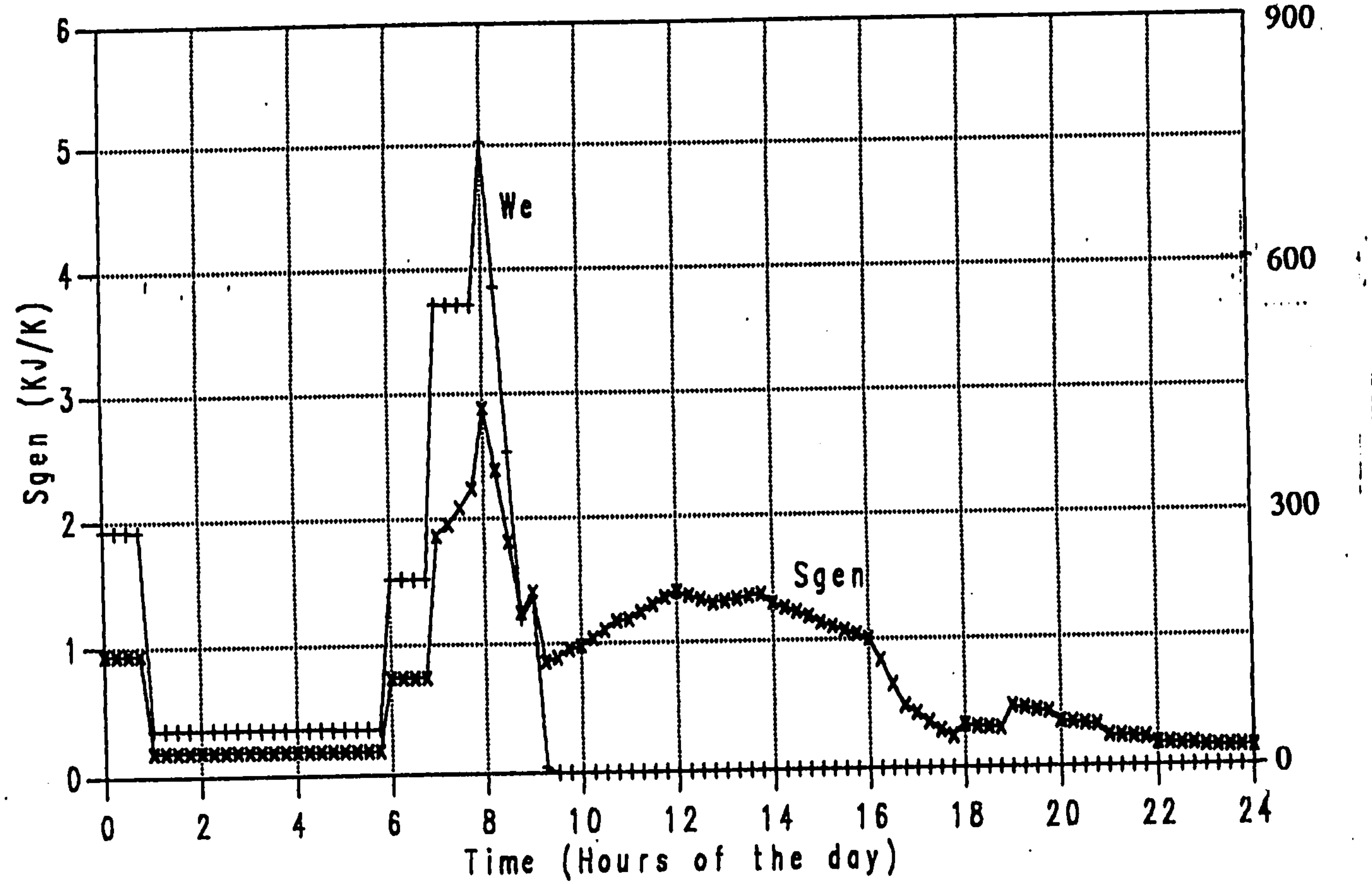



Fig. 4.6 Variation of Entropy and Auxiliary Energy (Vs/Ac = 50, M/Vs = 1.0, Rand Profile)

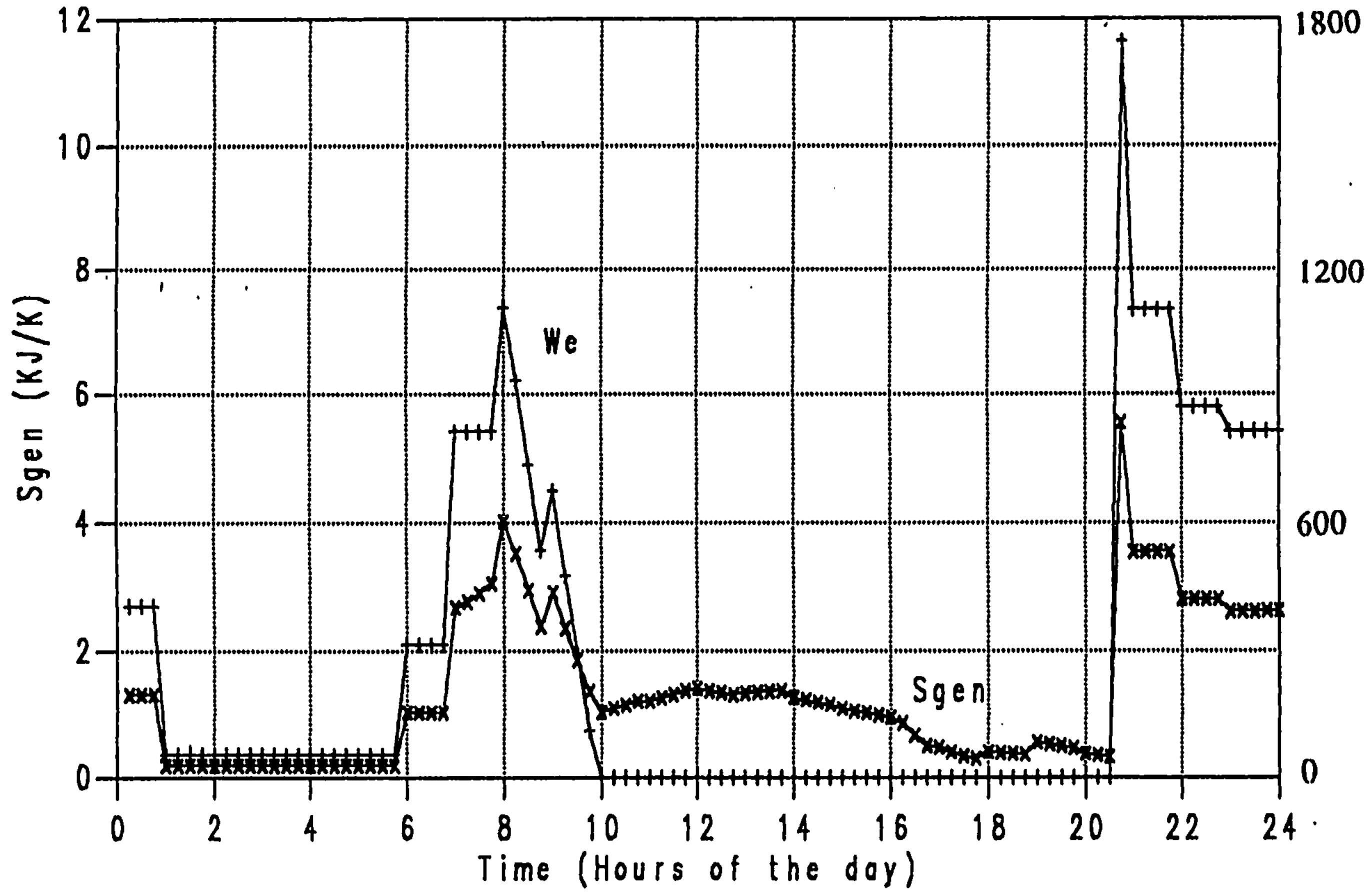



Fig. 4.7 Variation of Entropy and Auxiliary Energy
(Vs/Ac=50, M/Vs=1.5, Rand Profile)

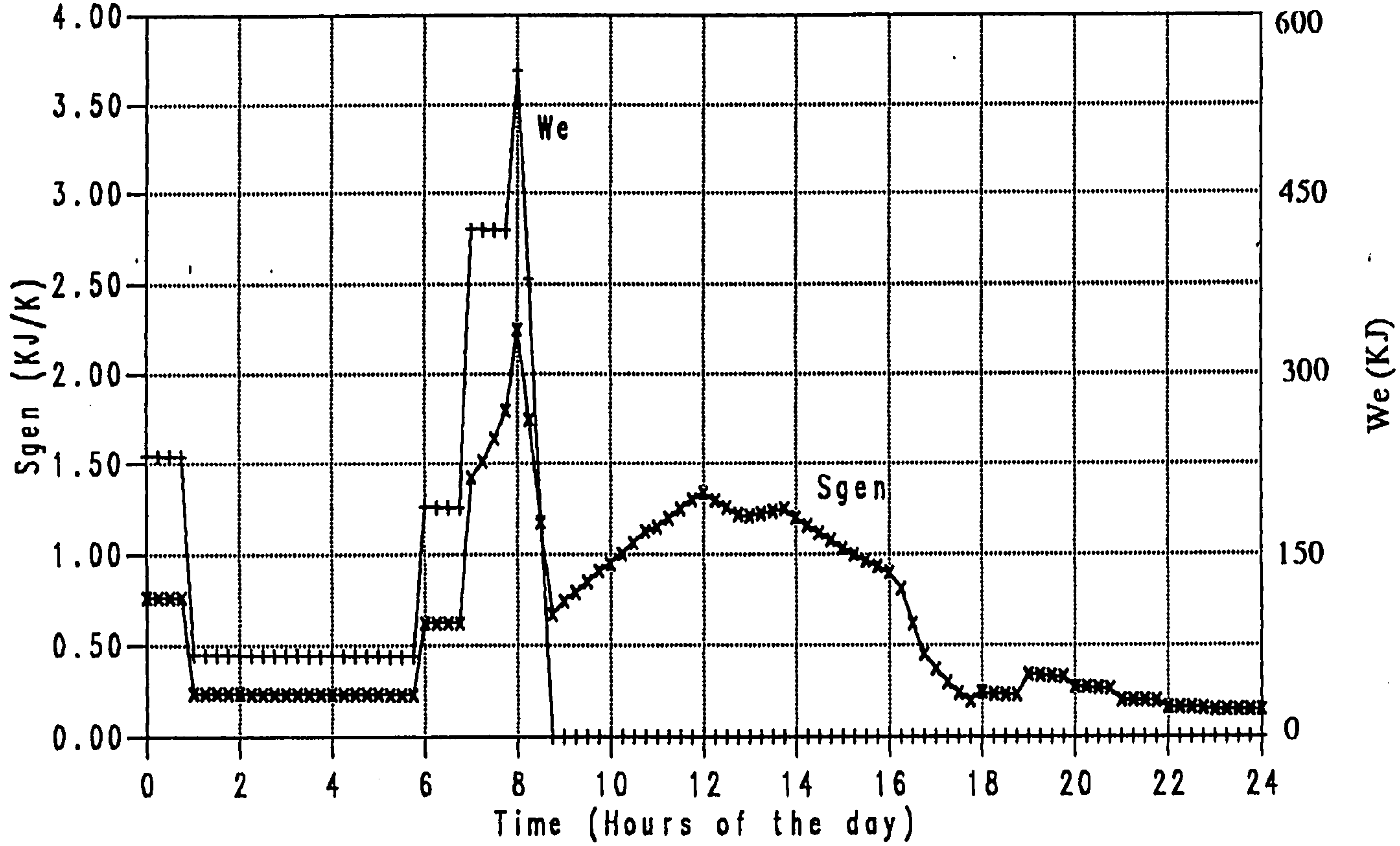



Fig. 4.8 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 70, M/Vs = 0.5, Rand Profile)

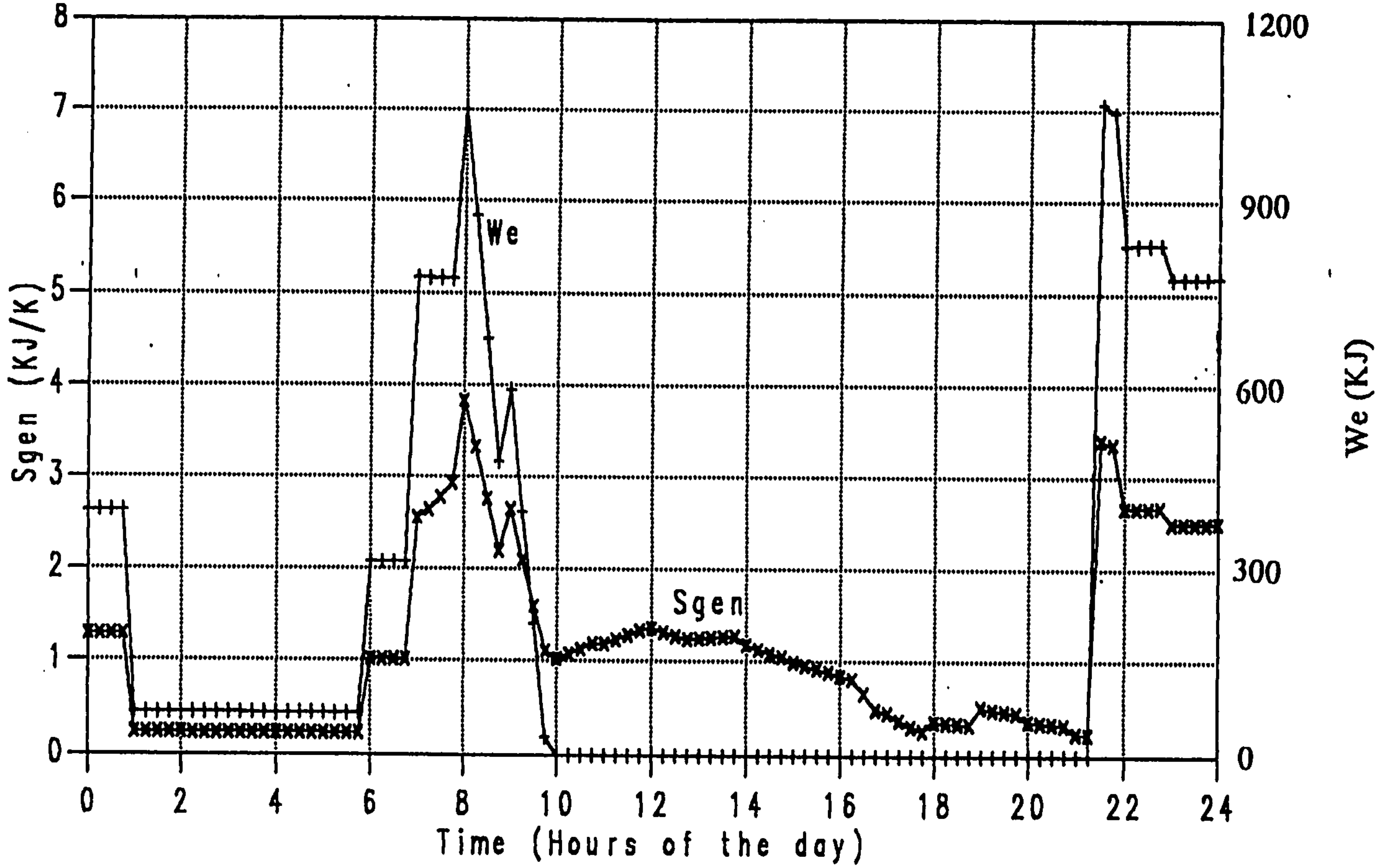



Fig. 4.9 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 70, M/Vs = 1.0, Rand Profile)

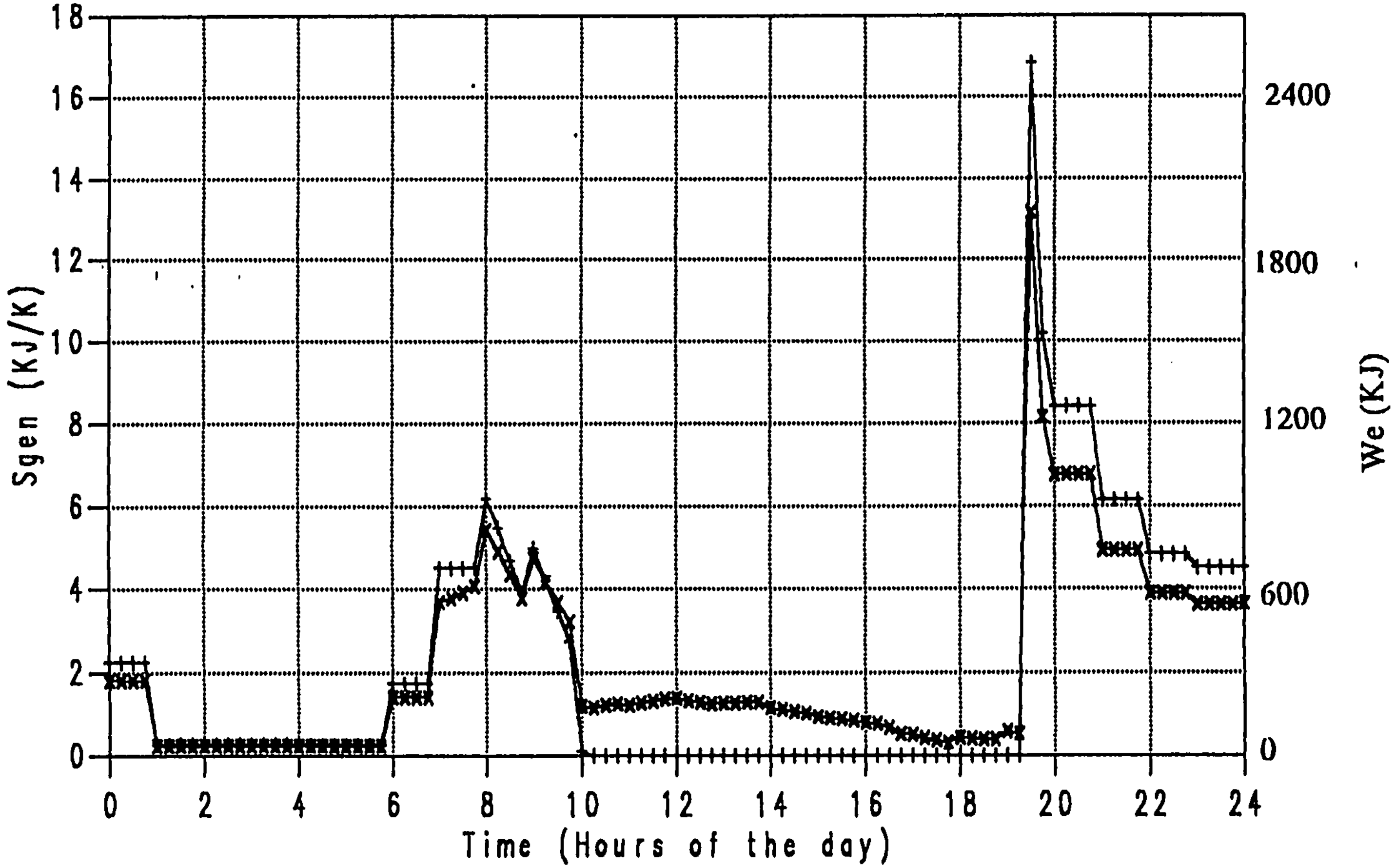



Fig. 4.10 Variation of Entropy and Auxiliary Energy (Vs/Ac = 70, M/Vs = 1.5, Rand Profile)

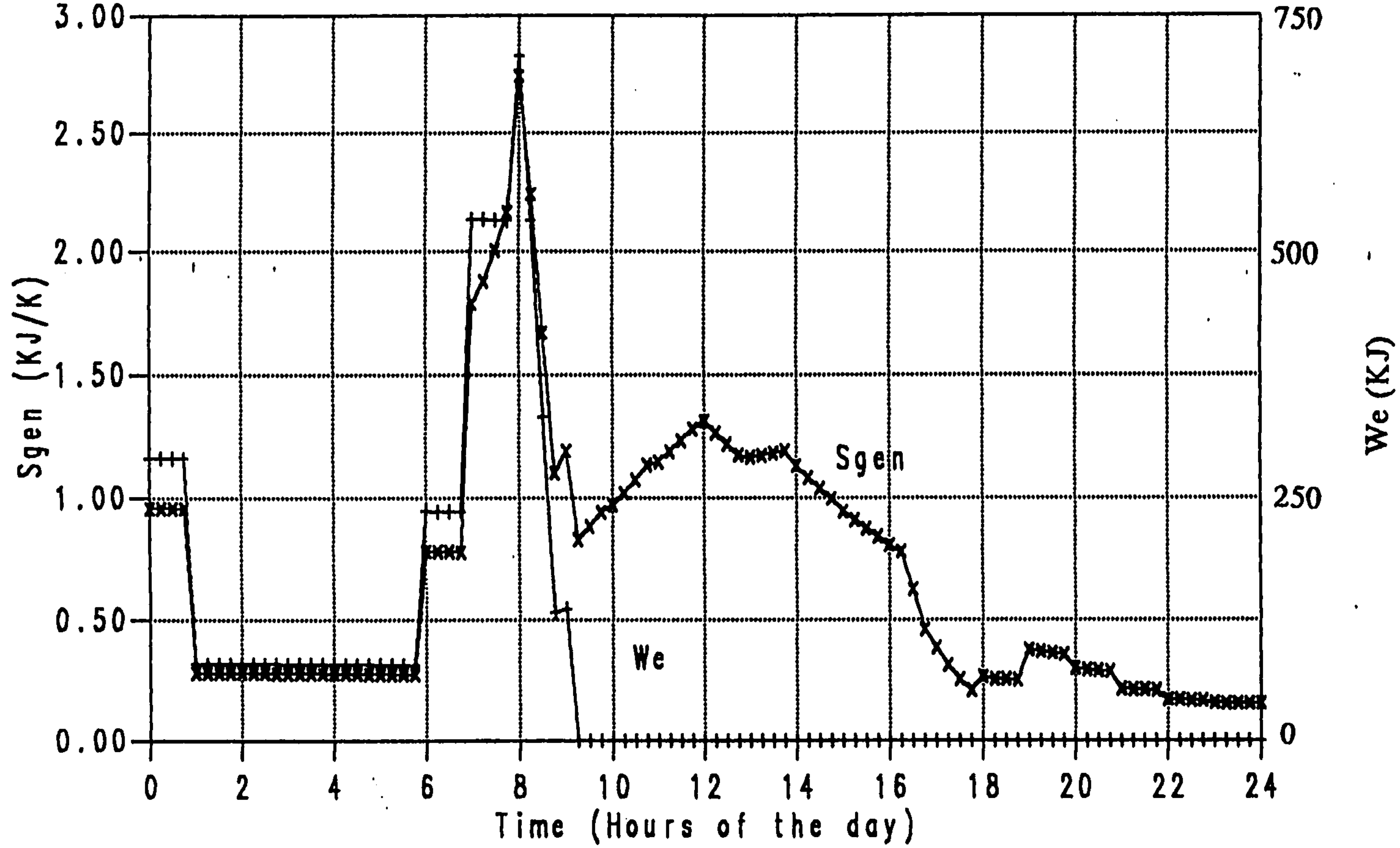



Fig. 4.11 Variation of Entropy and Auxiliary Energy
(Vs/Ac=90, M/Vs=0.5, Rand Profile)

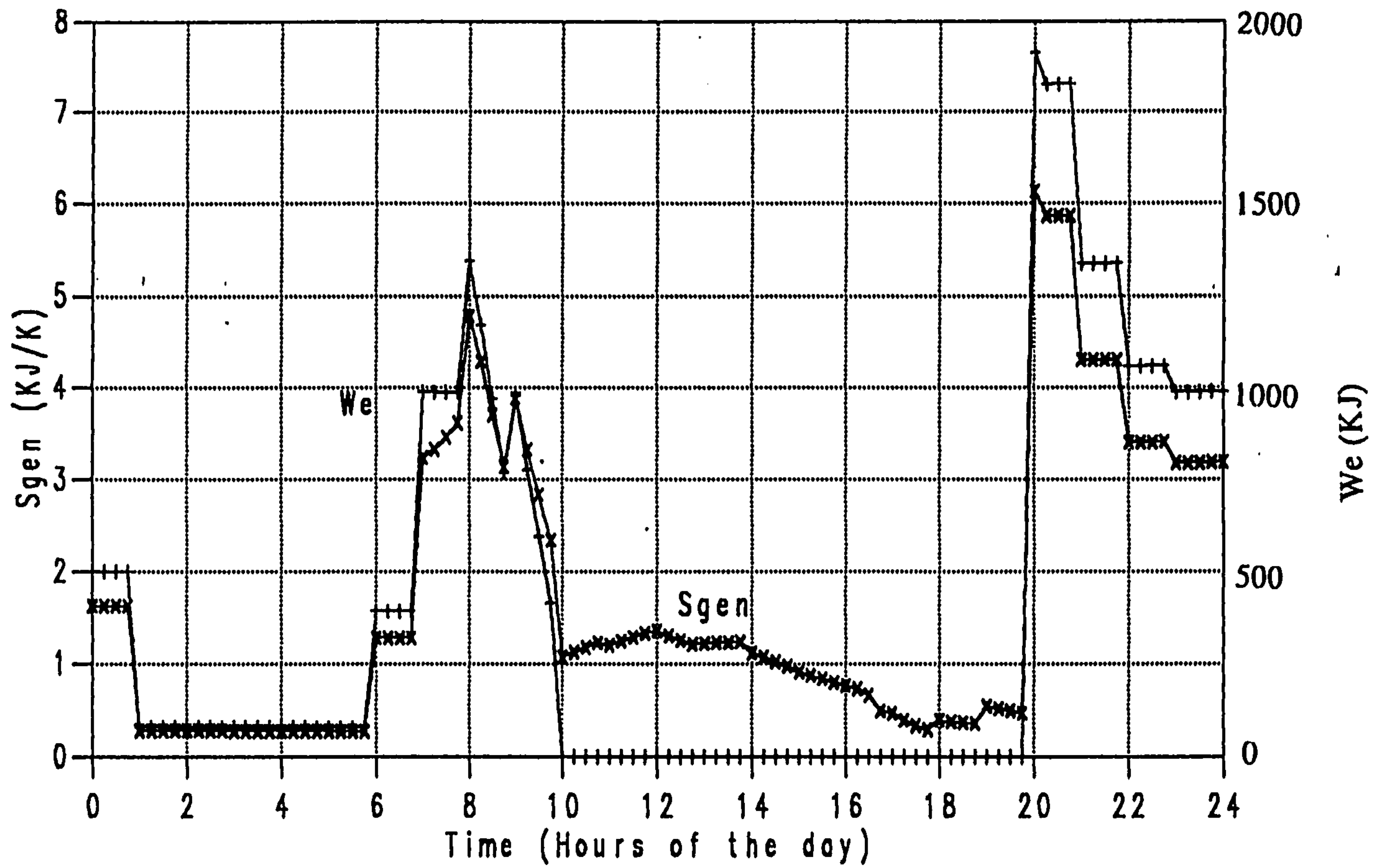



Fig. 4.12 Variation of Entropy and Auxiliary Energy
($Vs/Ac = 90$, $M/V_s = 1.0$, Rand Profile)

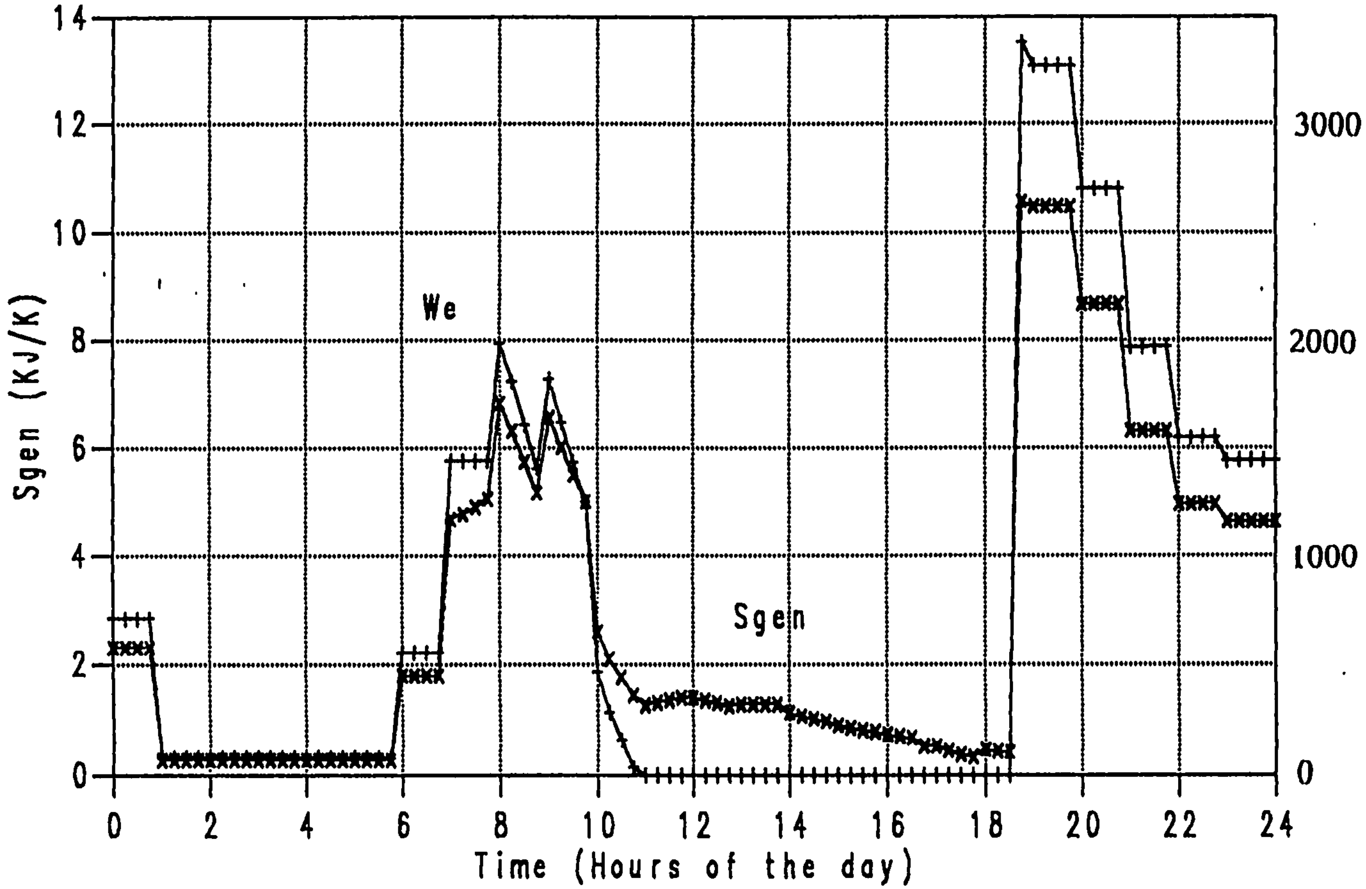



Fig. 4.13 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 90, M/Vs = 1.5, Rand Profile)

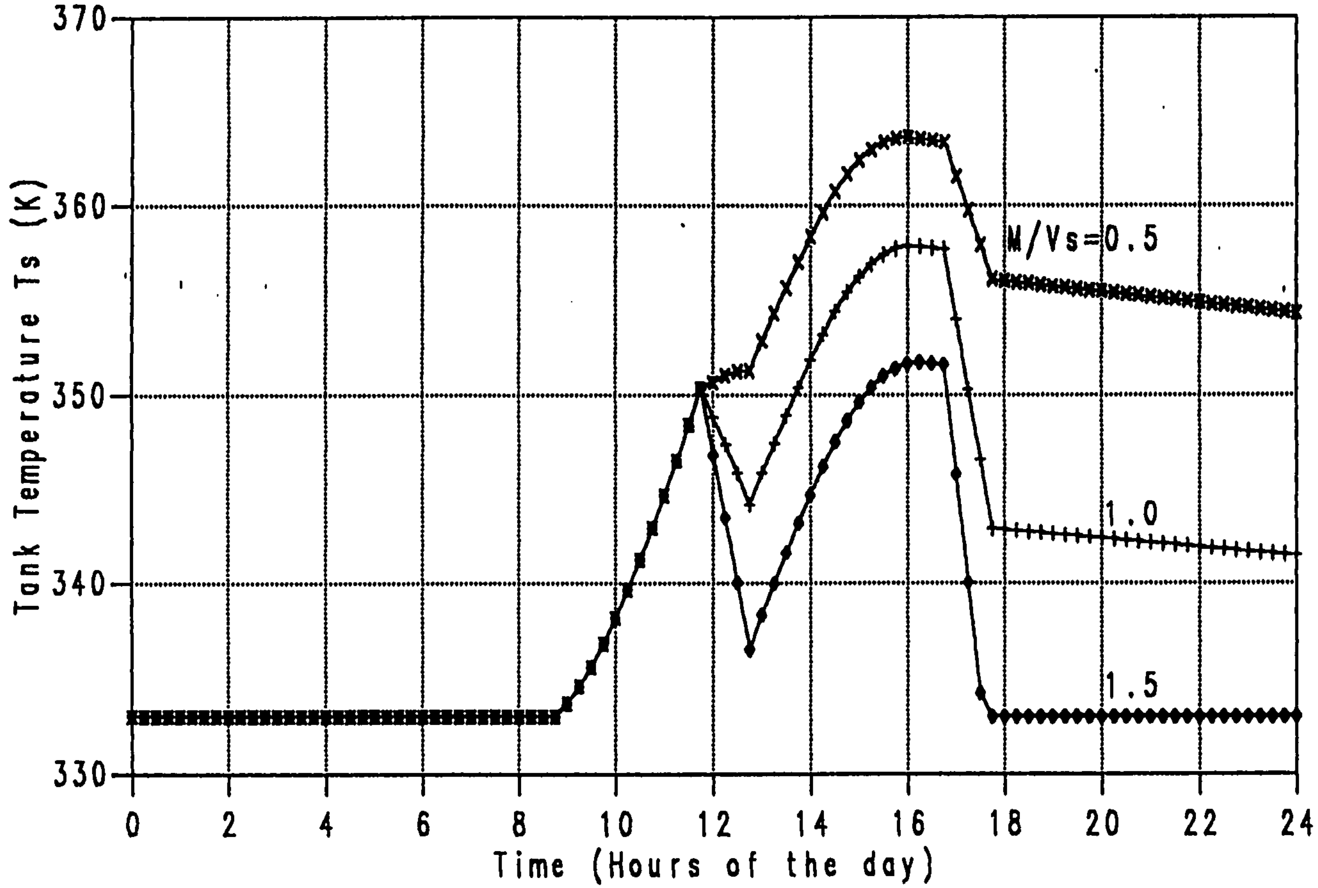



Fig. 4.14 Variation of Tank Temperature with Load
(Vs/Ac = 50, ASHRAE profile)

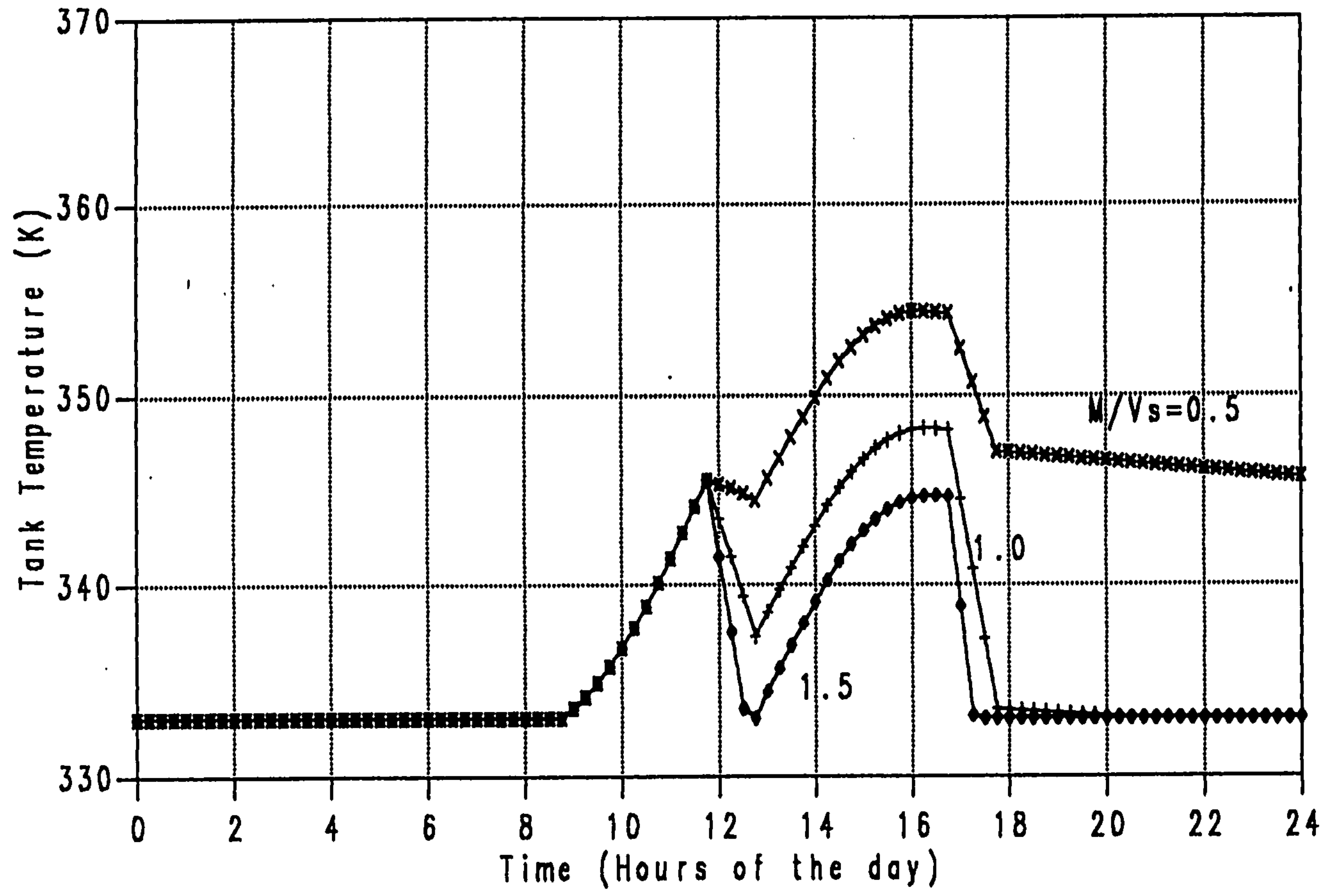



Fig. 4.15 Variation of Tank Temperature with Load
(Vs/Ac=70, ASHRAE profile)

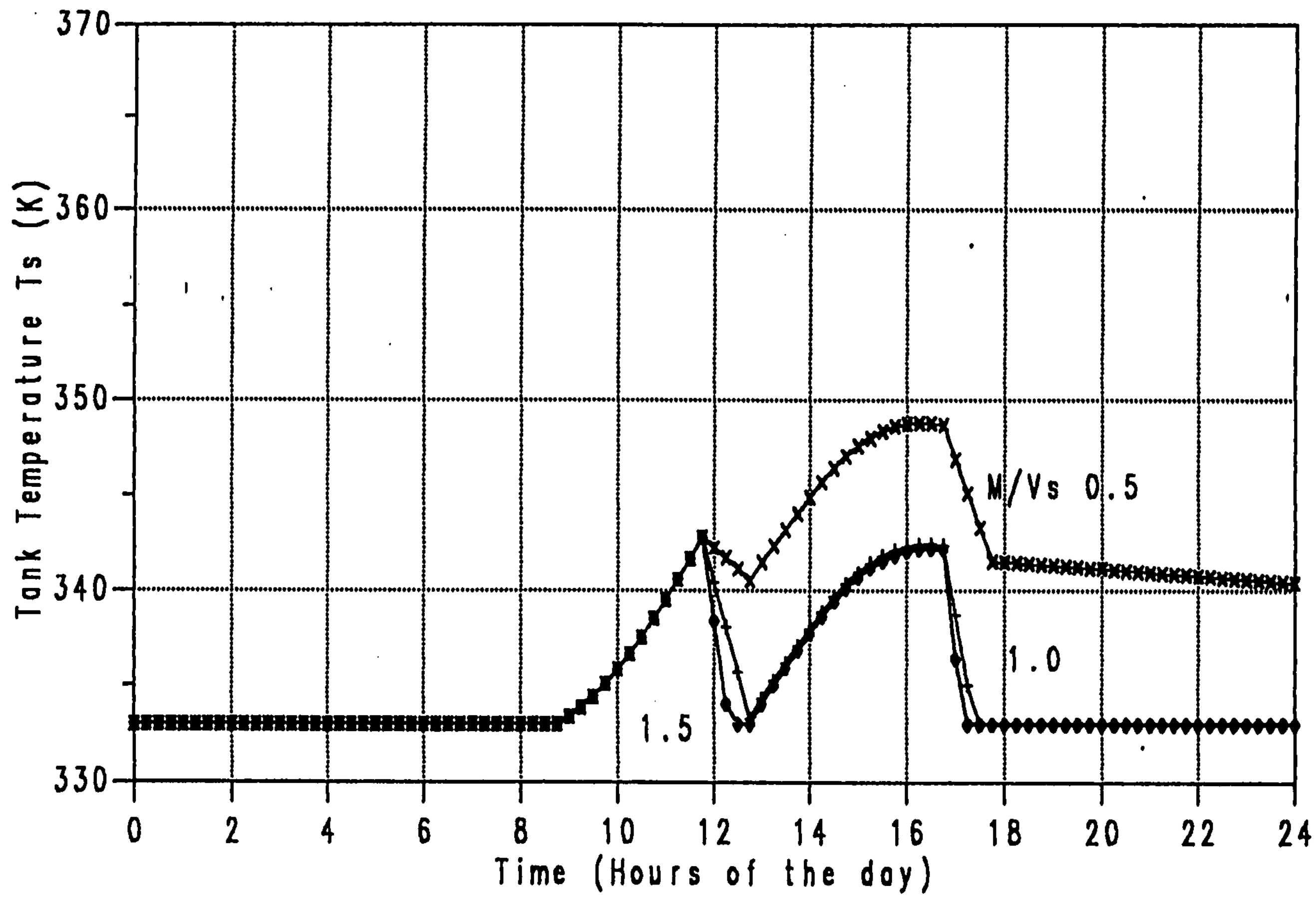



Fig. 4.16 Variation of Tank Temperature with Load
(Vs/Ac=90, ASHRAE profile)

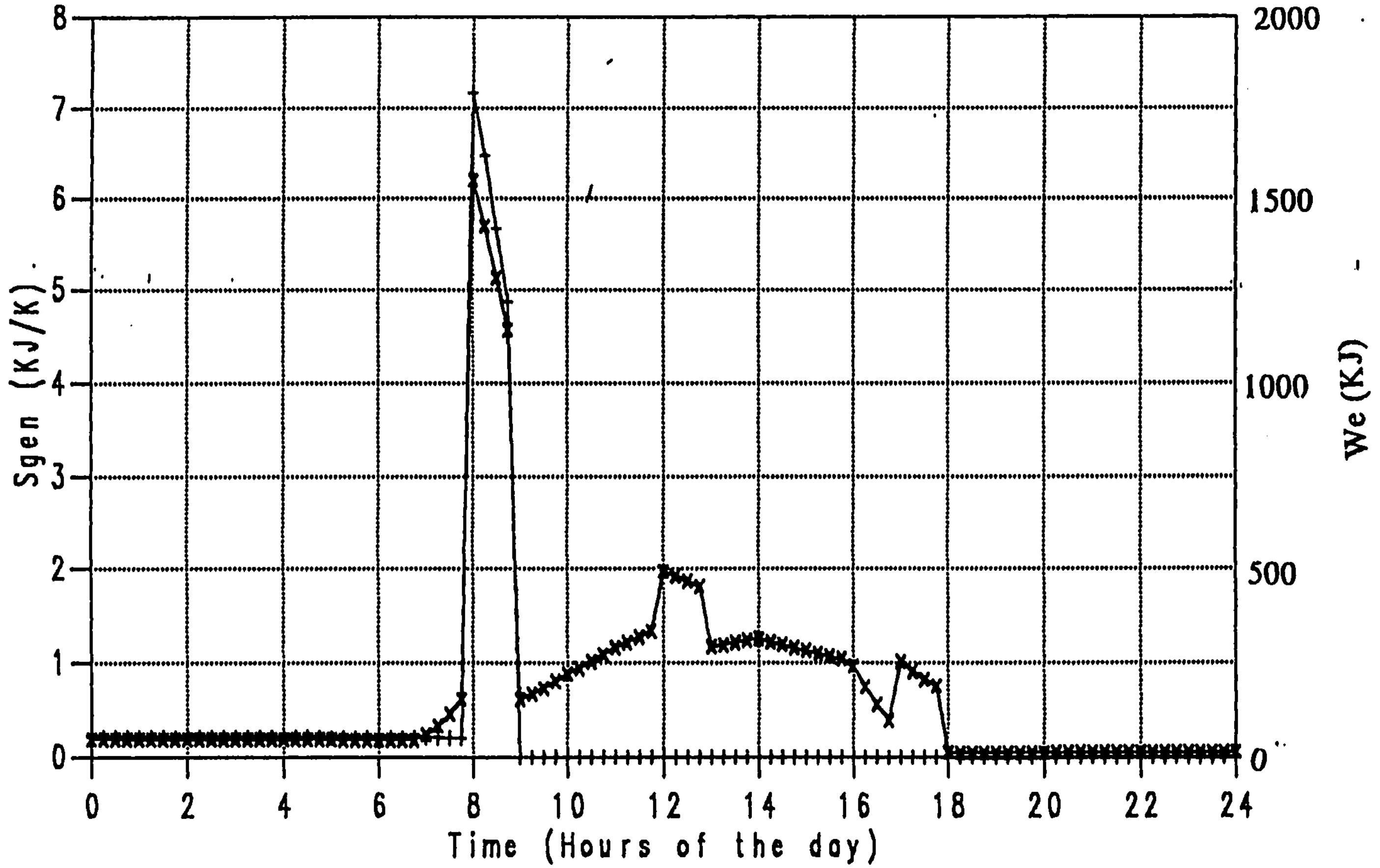



Fig. 4.17 Variation of Entropy and Auxiliary Energy
(Vs/Ac=50, M/Vs=0.5, ASHRAE Profile)

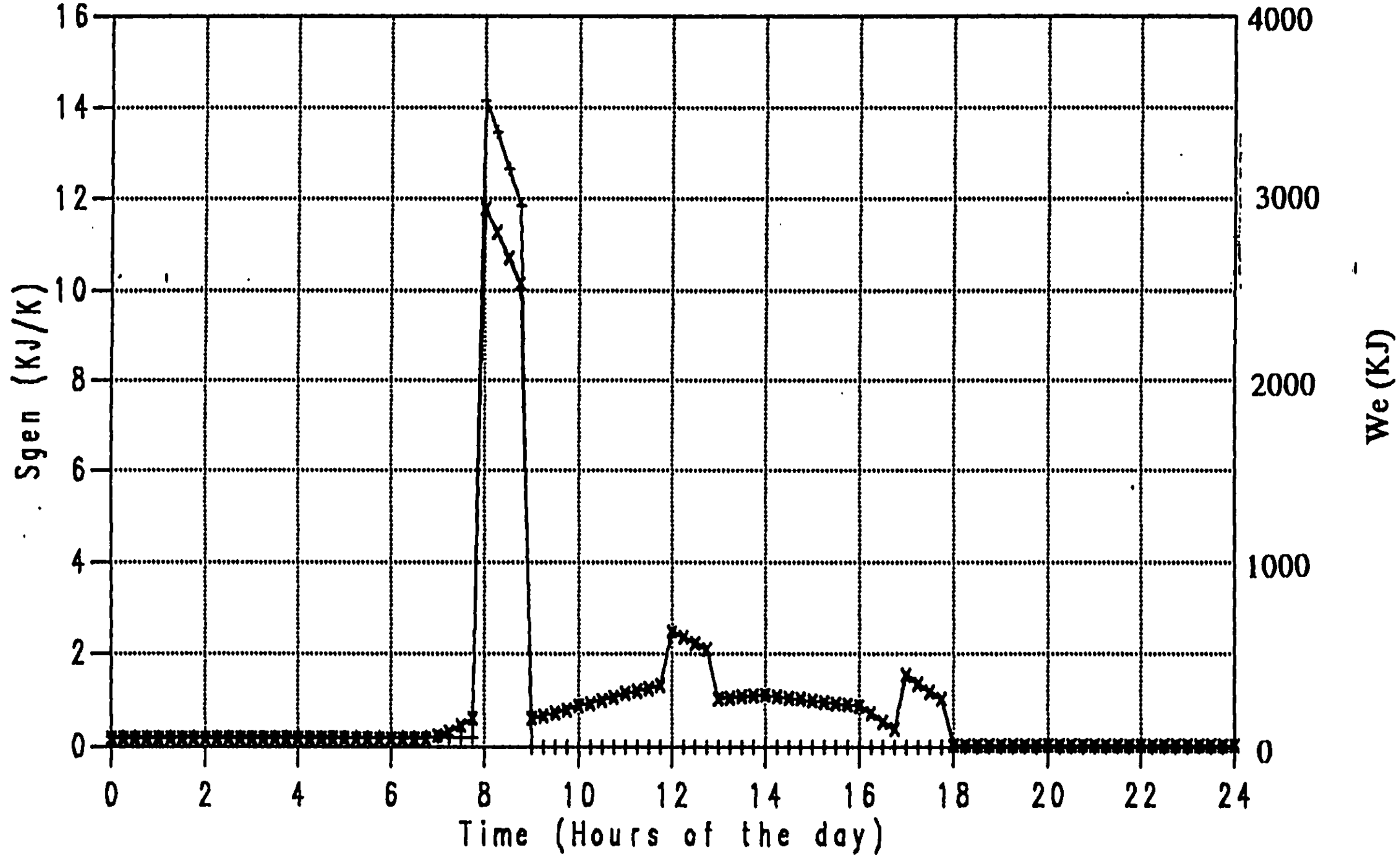



Fig. 4.18 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 50, M/Vs = 1.0, ASHRAE Profile)

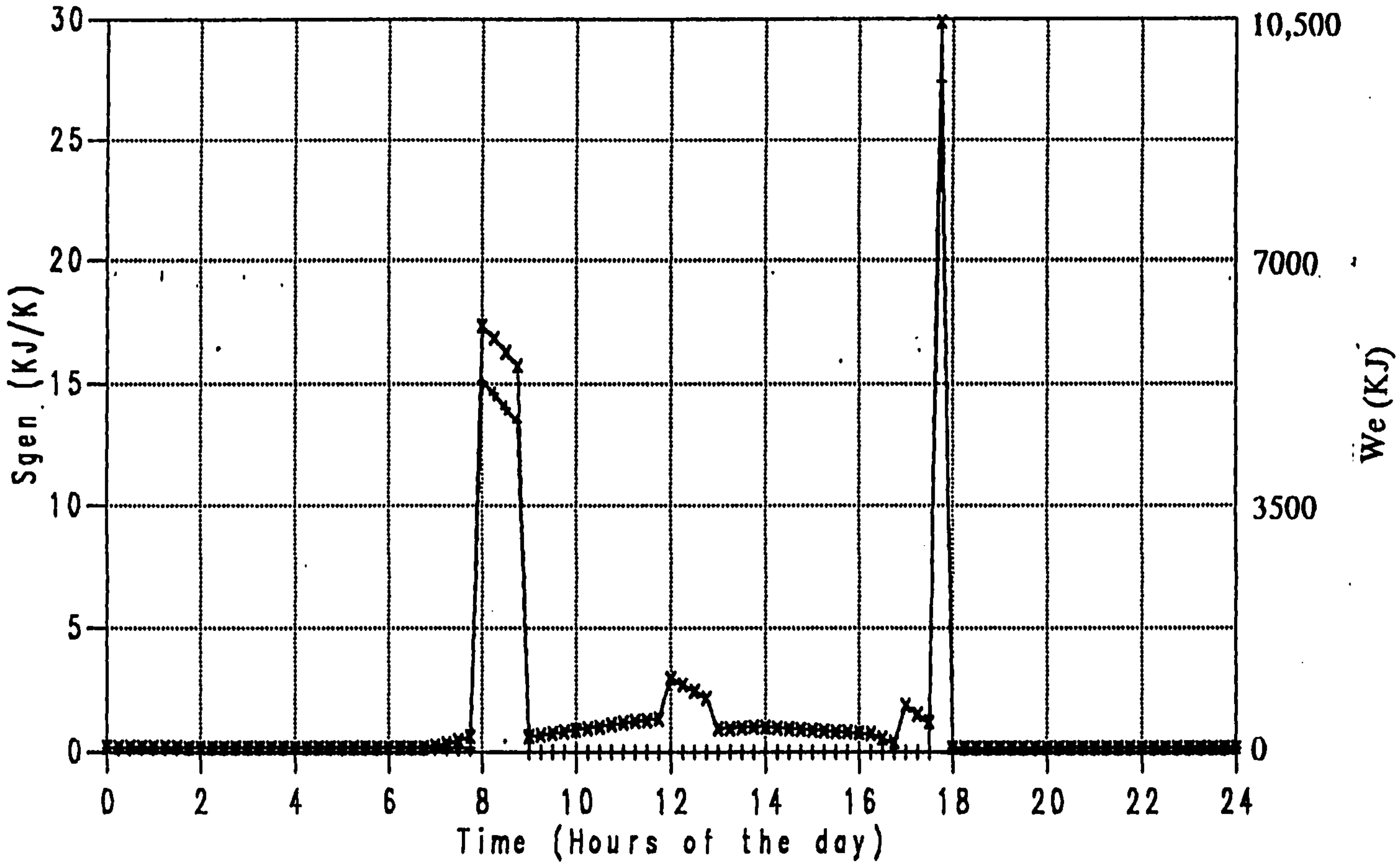



Fig. 4.19 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 50, M/Vs = 1.5, ASHRAE Profile)

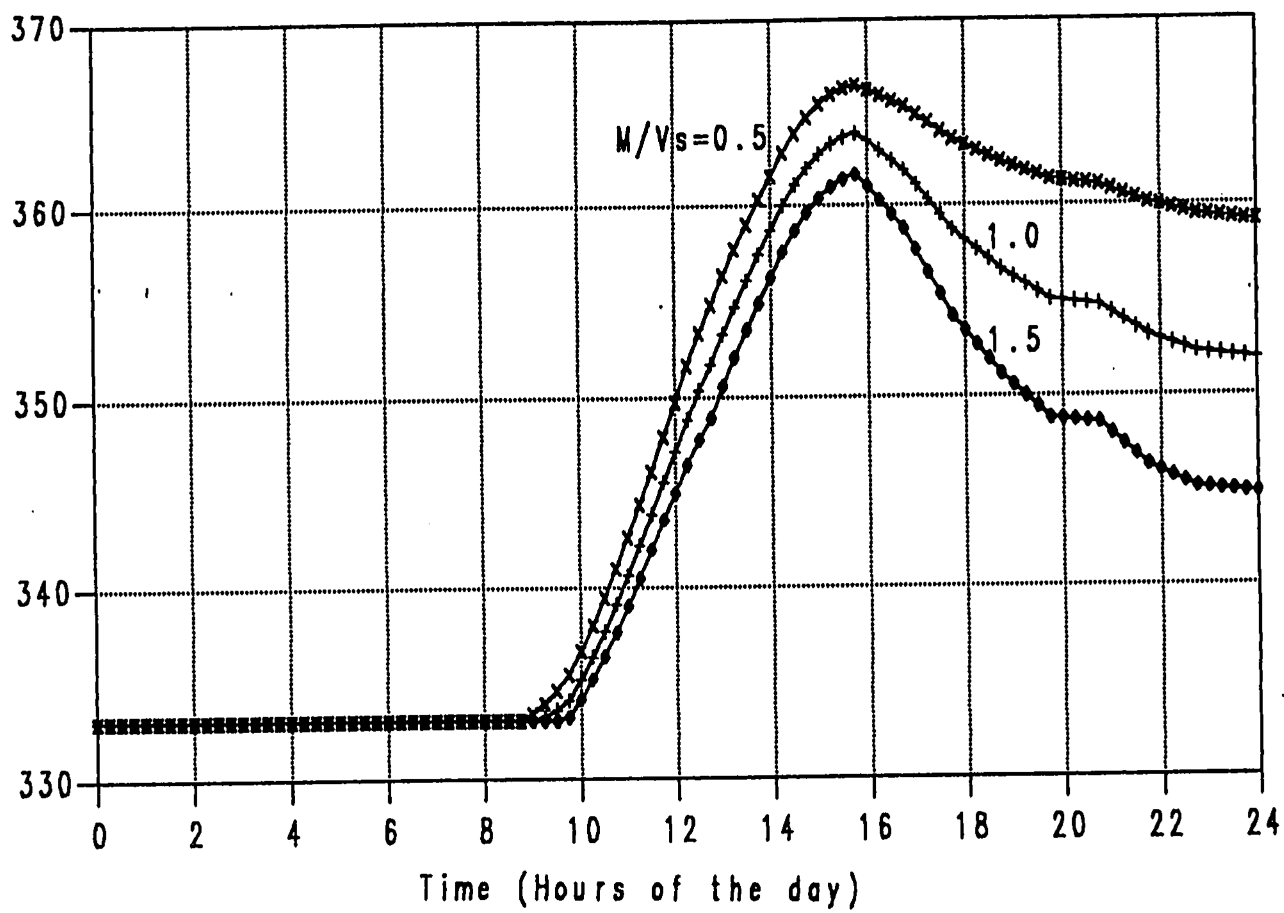



Fig. 4.20 Variation of Tank Temperature with Load
(Vs/Ac=50, Vitro profile)

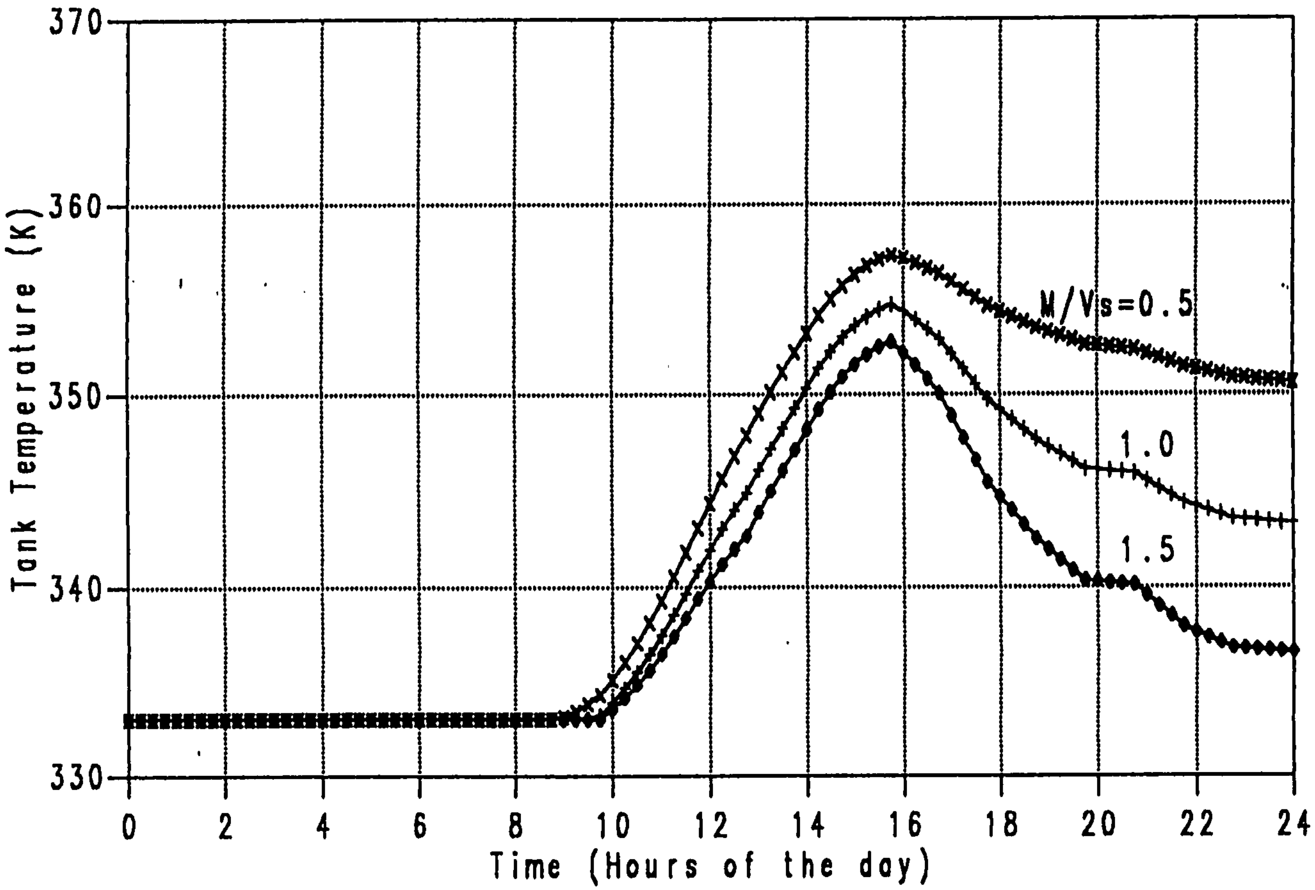



Fig. 4.21 Variation of Tank Temperature with Load
(Vs/Ac=70, Vitro profile)

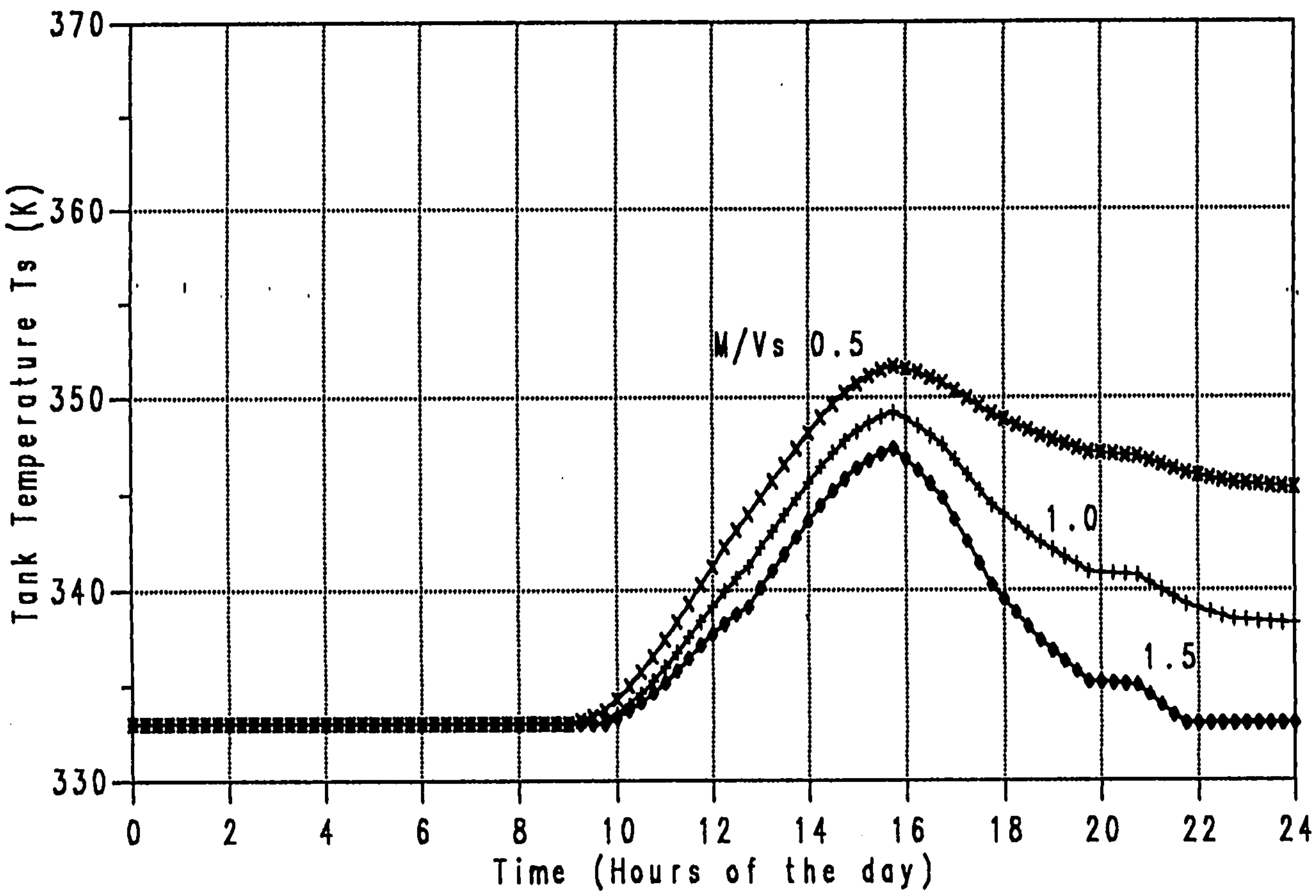



Fig. 4.22 Variation of Tank Temperature with Load
(Vs/Ac=90, Vitro profile)

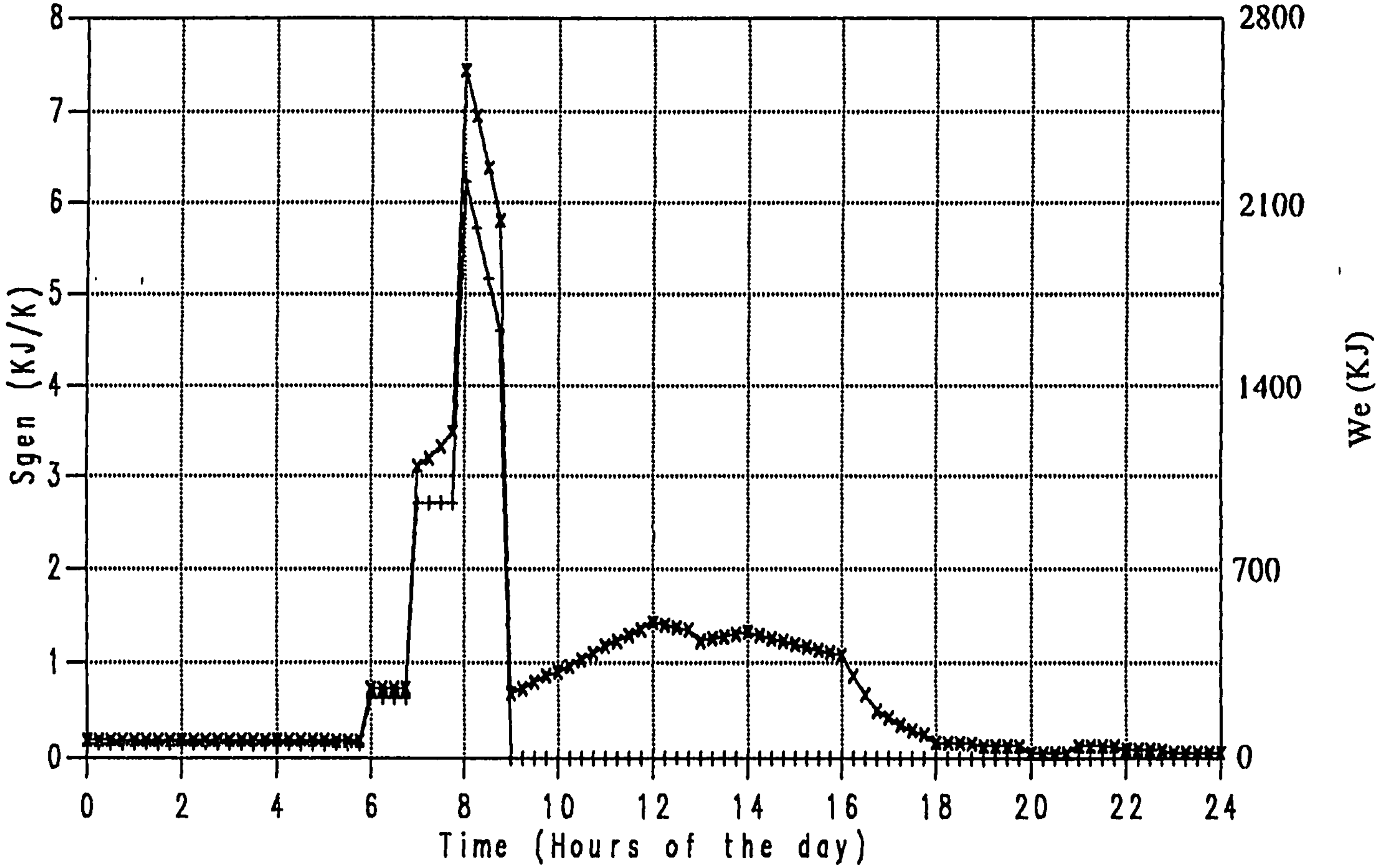



Fig. 4.23 Variation of Entropy and Auxiliary Energy
(Vs/Ac=50, M/Vs=0.5, Vitro Profile)

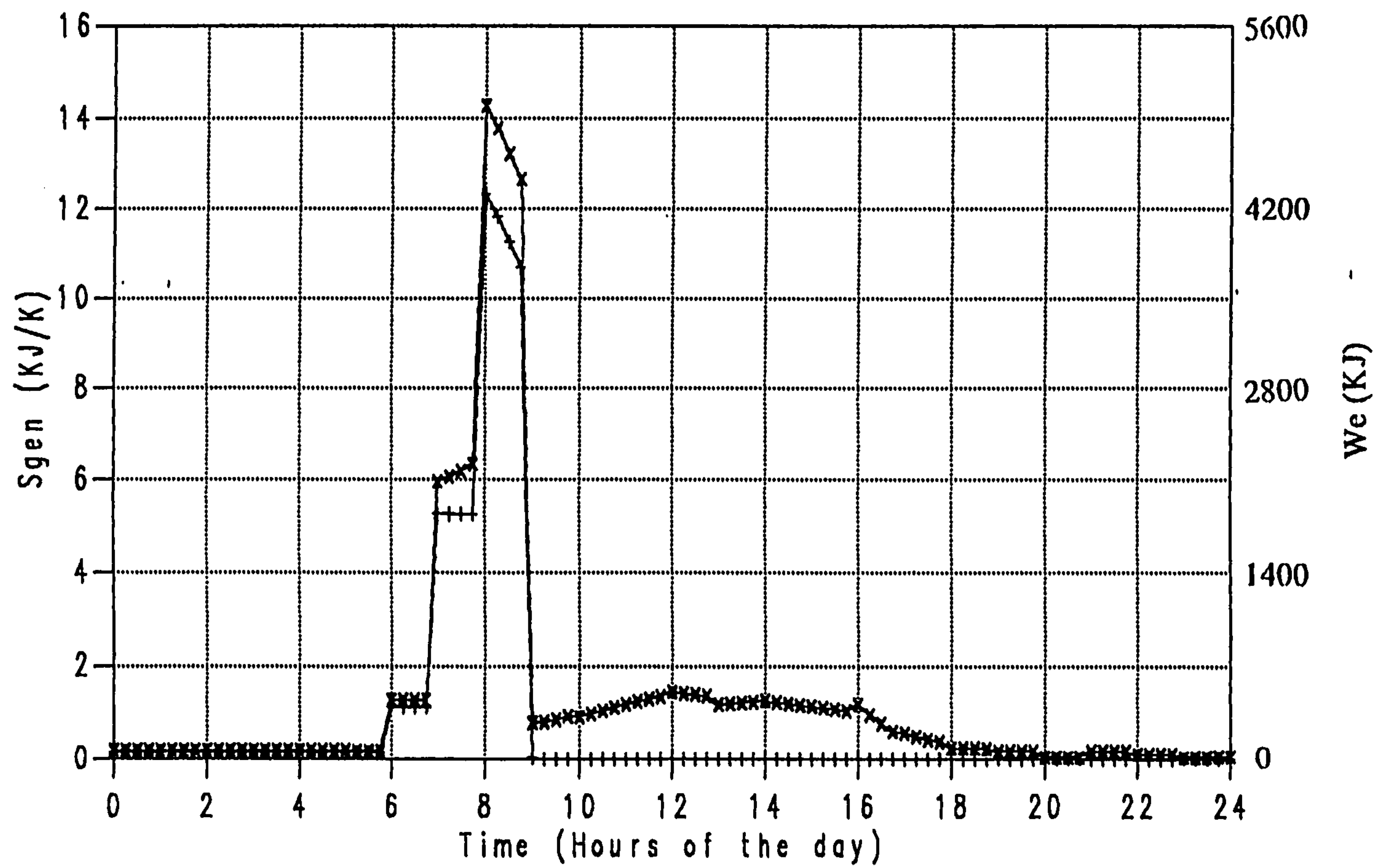



Fig. 4.24 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 50, M/Vs = 1.0, Vitro Profile)

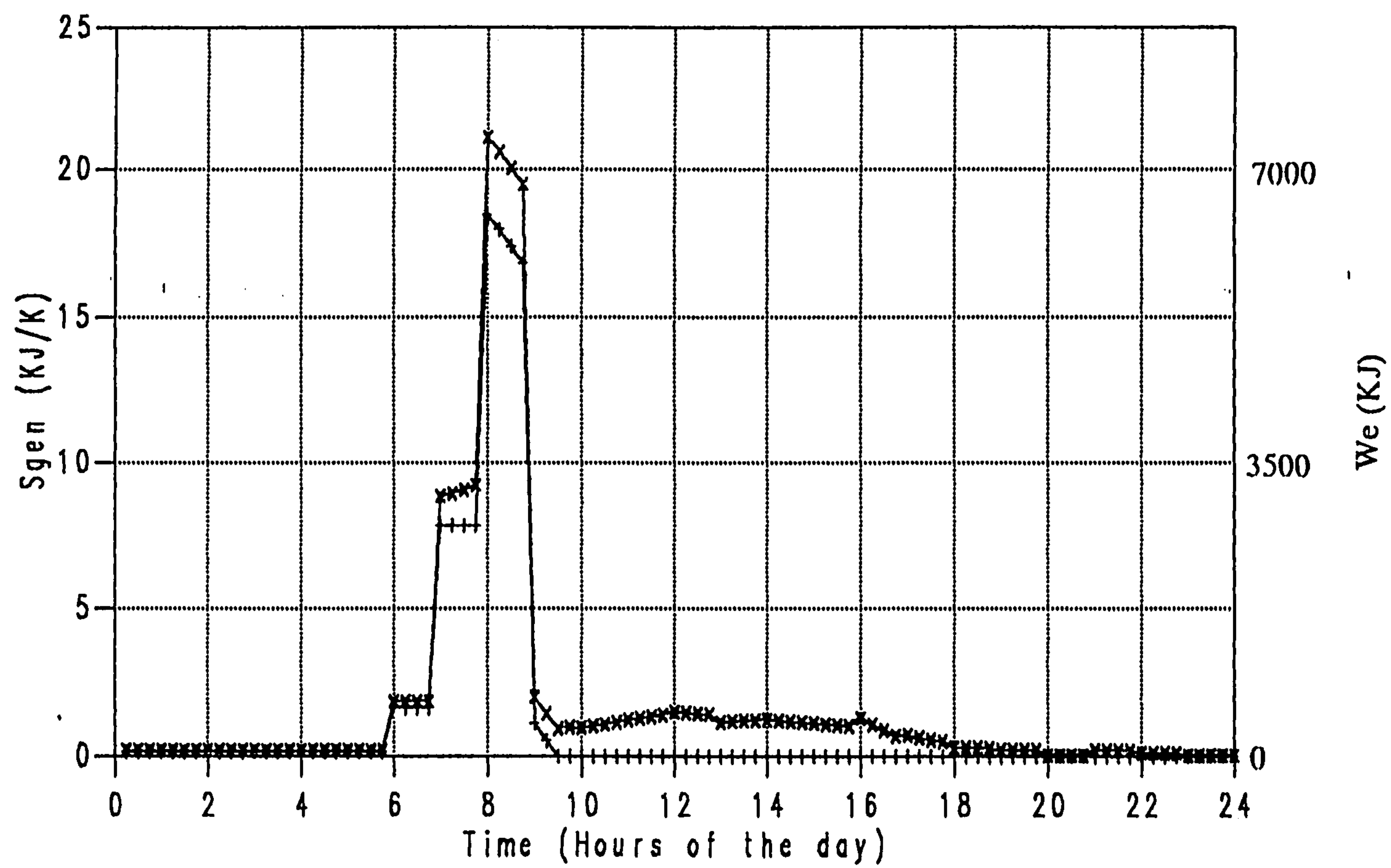



Fig. 4.25 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 50, M/Vs = 1.5, Vitro Profile)

TABLE (4.2) Total Auxiliary Energy Consumption and Total Entropy Generation for System#1

| Case | $V_s$ (lts) | $V_s/A_c$ | $M/V_s$ | Rand Profile S (Total) (KJ/°K) | Rand Profile Aux.Heat Total(KJ) | ASHRAE Profile S (Total) (KJ/°K) | ASHRAE Profile Aux.Heat Total(KJ) | Vitro Profile S (Total) (KJ/°K) | Vitro Profile Aux. Heat Total(KJ) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 250 | 50 | 0.5 | 58.52 | 4013.5 | 62.20 | 7691.4 | 86.58 | 13519.8 |
| 2 | " | " | 1.0 | 70.68 | 7183.1 | 92.86 | 14675.4 | 128.90 | 26343.0 |
| 3 | " | " | 1.5 | 127.17 | 25346.7 | 147.44 | 32522.4 | 172.70 | 39706.2 |
| 4 | 350 | 70 | 0.5 | 62.04 | 5577.8 | 77.99 | 10951.2 | 102.60 | 19107.0 |
| 5 | " | " | 1.0 | 110.35 | 20460.6 | 113.52 | 21698.1 | 163.20 | 37486.8 |
| 6 | " | " | 1.5 | 202.89 | 50013.9 | 227.60 | 57190.5 | 244.31 | 64001.7 |
| 7 | 450 | 90 | 0.5 | 67.44 | 7369.92 | 87.78 | 14166.0 | 119.7 | 24651.0 |
| 8 | " | " | 1.0 | 163.21 | 37558.8 | 183.17 | 44496.4 | 199.98 | 49083.3 |
| 9 | " | " | 1.5 | 278.37 | 74139.3 | 294.80 | 78454.8 | 340.21 | 94491.0 |

Vitro profile, and early morning hours and late afternoon hours in ASHRAE profile is the fundamental reason.

2. Although in Table (4.2), total entropy generated and total auxiliary energy required for various tank sizes and mass flows are evaluated, it is difficult to see the degree of their effect on the resulting entropy generation and auxiliary energy consumption. In order to see the variation of entropy generation and auxiliary heat consumption per kg of water drawn for a certain tank volume, the total entropy generated and auxiliary energy needed per kg of water are plotted against the total mass flow for various tank volumes in Figures 4.26 and 4.27 respectively, for Rand profile. The figures indicate that for small tank volumes, the minimum entropy generation and auxiliary energy consumption correspond to a total mass flow which is approximately equal to the tank volume. However the minimum shifts towards lower mass flow to tank volume ratios for larger tank volumes, but then the amount of water drawn is much less than tank size.

In practice, tank volume to collector area ratio is taken to be in the range of 50 to 75. It is clear from Fig.(4.26) which is obtained by the second law analysis that in the above tank size to collector area range, for minimum entropy generation the total water withdrawal should be approximately equal to the tank volume. However, it follows from Fig.(4.27) which is obtained by the first law energy balance that for a tank volume to collector area ratio of 50, the minimum auxiliary energy consumption indicates that the total water withdrawal should be approximately equal to the tank volume. But for a ratio of 70 water withdrawal must be much less than the tank volume. It can be concluded from both figures that a tank volume to collector area ratio of 50 provides the optimum for

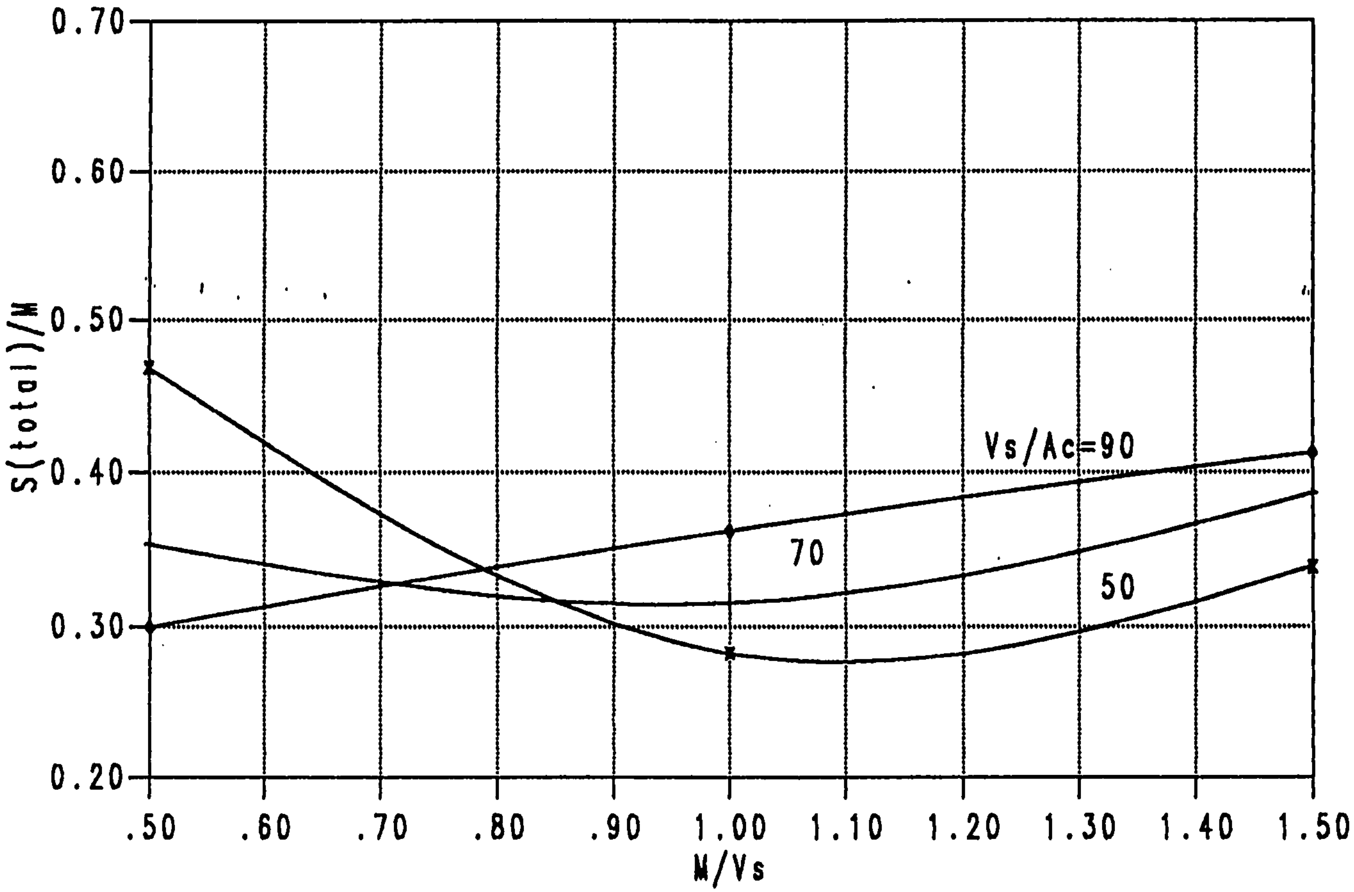



Fig. 4.26 Total Entropy generation per liter of water withdrawal (Rand profile)

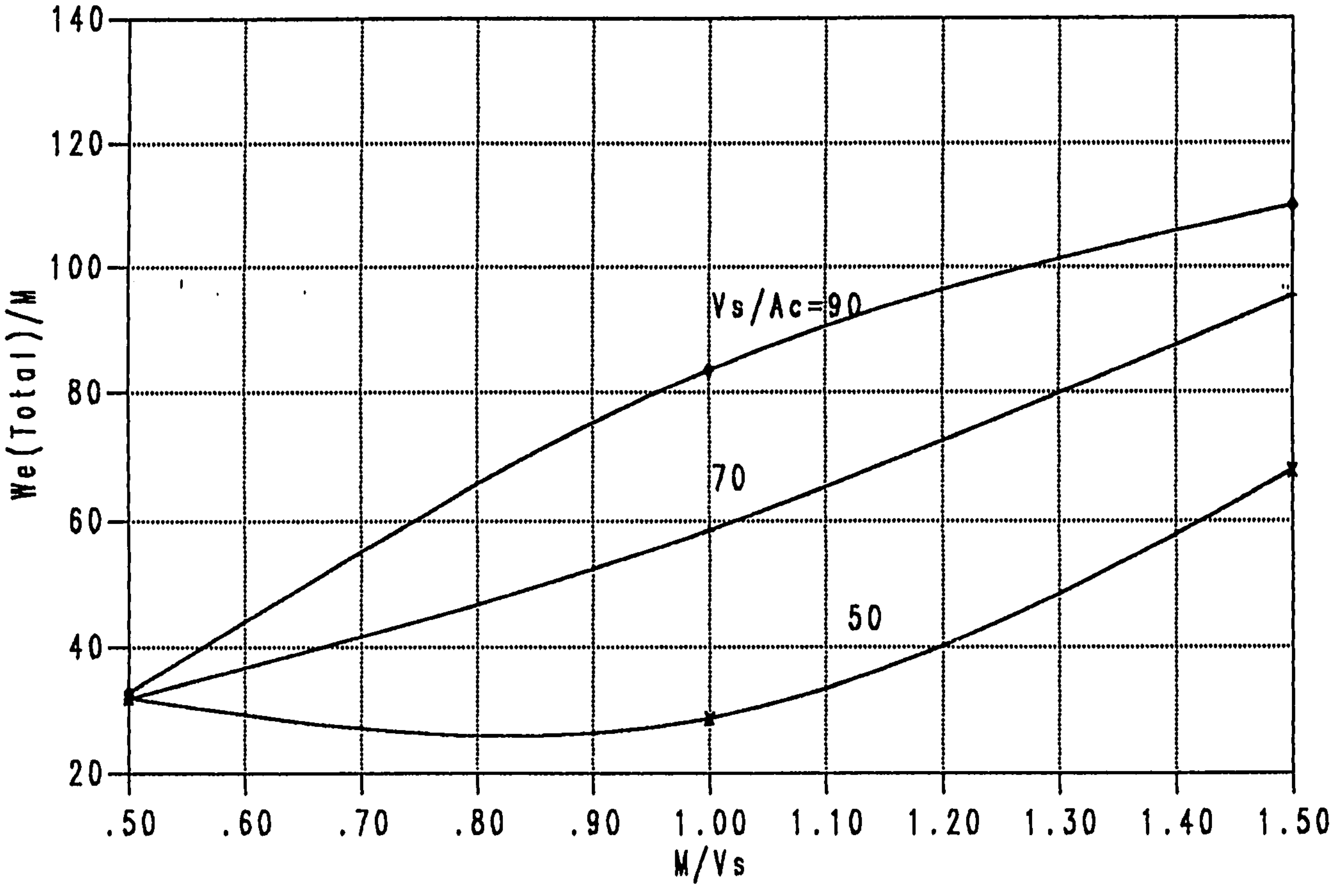



Fig. 4.27 Total Auxiliary energy per liter of water withdrawal (Rand profile)

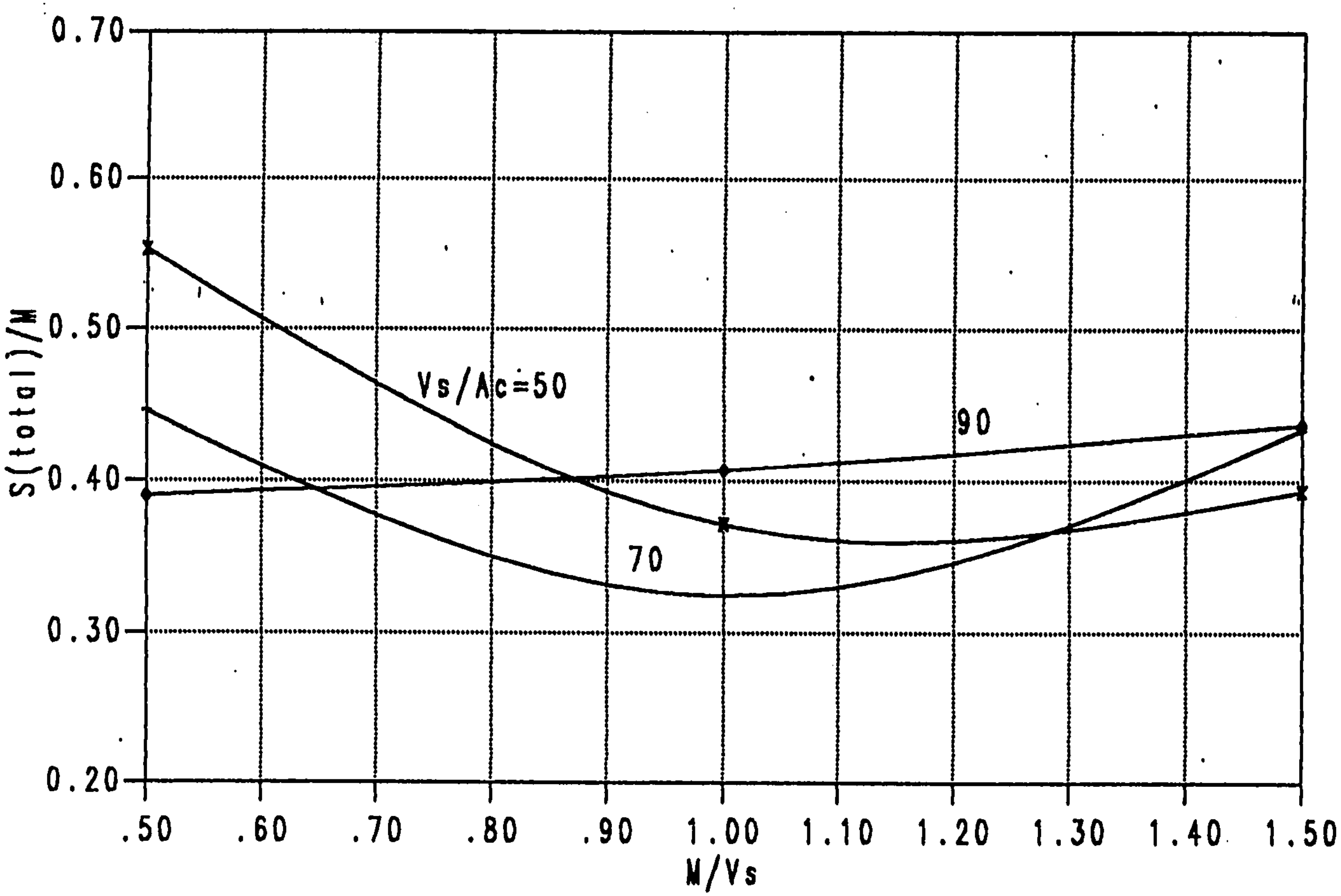



Fig. 4.28 Total Entropy generation per liter of water withdrawal (ASHRAE profile)

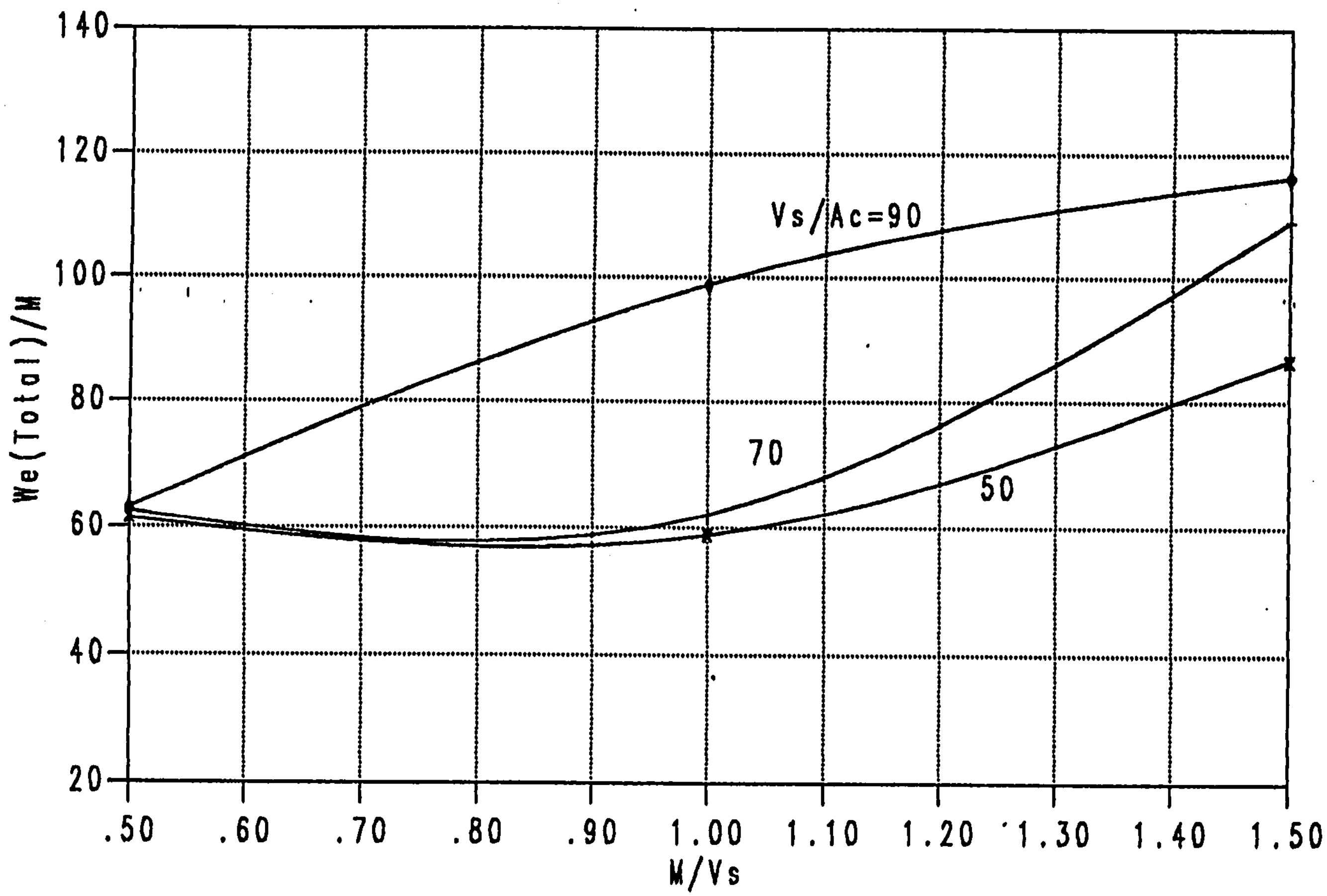



Fig. 4.29 Total Auxiliary energy per liter of water withdrawal (ASHRAE profile)

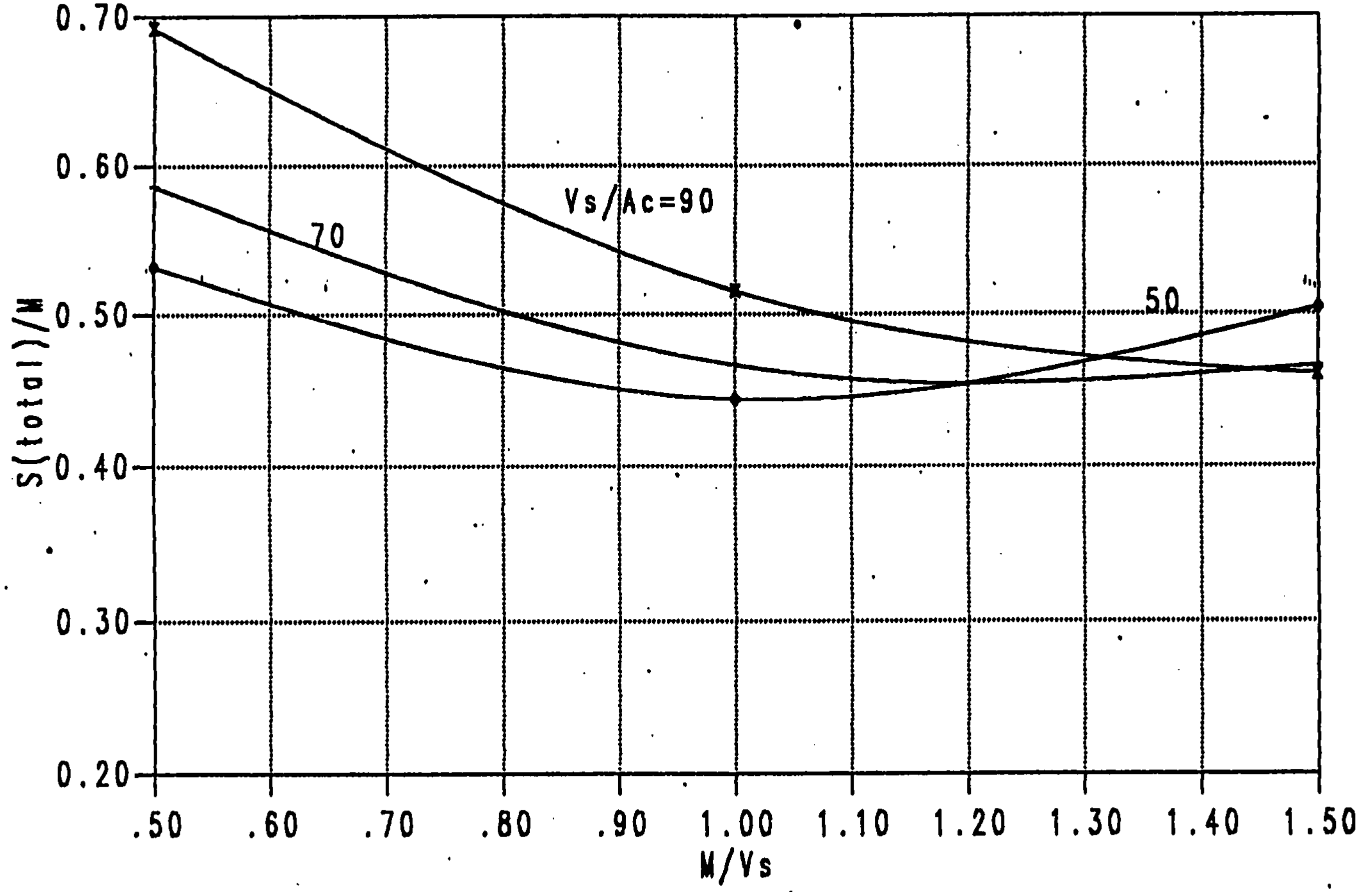



Fig. 4.30 Total Entropy generation per liter of water withdrawal (Vitro profile)

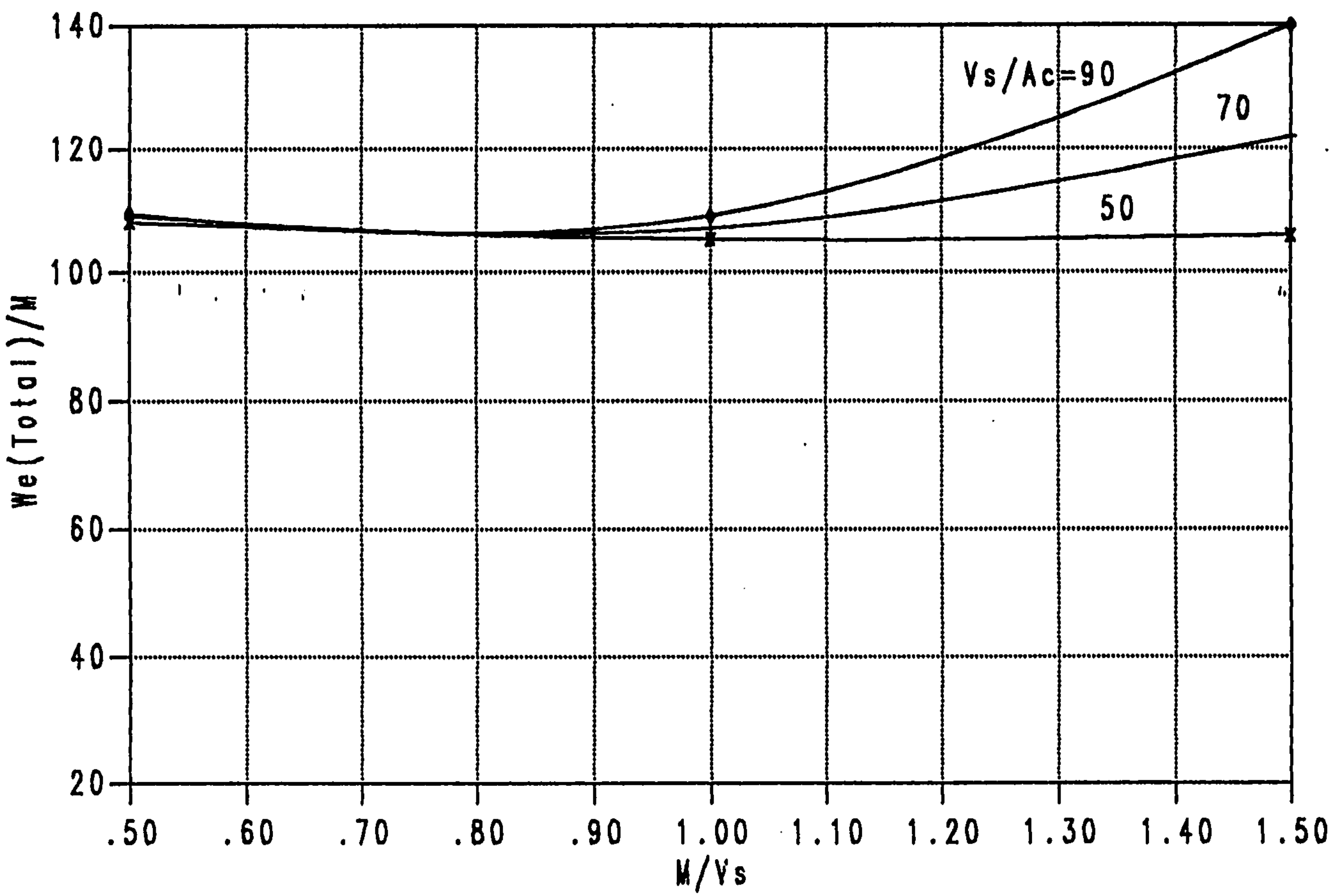



Fig. 4.31 Total Auxiliary energy per liter of water withdrawal (Vitro profile)

both tank volume and water withdrawal.

The plots of minimum total entropy generated per kg of water drawn using ASHRAE and Vitro profiles are presented in Figures 4.28 and 4.30, respectively. They indicate that for small tank volumes, the minimum total entropy generated corresponds to total load mass flow rates which are approximately equal to the tank volume. Again the minimum shifts to lower load mass flow to tank volume ratios for larger tank volumes. Similar behavior is observed for the total auxiliary energy per kg of water drawn as shown in Figures 4.29 and 4.31 for ASHRAE and Vitro profiles respectively.

From the above discussion, it can be concluded that for minimum entropy generation and minimum auxiliary energy consumption, the hot water withdrawn should be approximately equal to tank volume regardless of the load profile used.

On the whole the Rand profile creates less entropy per kg of hot water withdrawal while requires less auxiliary energy per kg of water, irrespective of tank size and hot water withdrawal.

### 4.3 Simulation of system#2

This section is concerned with the simulation study of the water heating system#2 shown in Fig.(3.2) subject to three hourly load profiles (Rand, ASHRAE and Vitro) having the same total daily load. In this system the backup heater is in series with the storage system. If the temperature of water coming from storage tank is less than the desired load temperature $T_D$, then the water is heated up in the backup heater until the temperature reaches $T_D$. On the other hand if the temperature of water from storage tank is greater than the desired load temperature then it is reduced to $T_D$ by mixing with cold water from mains in a mixing valve.

The governing equations for the system#2, as taken from Chapter 3, are given below:

Dynamic equation of the tank from Eq.(3.13):

$$M_s C_s \frac{dT_s}{dt} = A_c F_R \left[ Q_{sol} - U_L (T_s - T_a) \right] - Q_{Loss} - Q_{Load} \qquad (4.3)$$

Where

$Q_{Loss} = (UA)_s (T_s - T_a)$ Heat lost to the ambient

$Q_{Load} = m_L C_p (T_s - T_L)$ Load on the system

$M_s$ Mass of the water in the storage tank (Kg)

$C_p$ Water specific heat (J/kg K)

Auxiliary energy equation:

$$W_E = m_L C_p (T_D - T_s) \tag{4.4}$$

Entropy equations from Eqns.(3.9, 3.20 and 3.24) are given as:

**Entropy equation for the collector:**

$$S_c = \frac{-I_T(\tau\alpha)A_c}{T_p} + \frac{A_c U_L (T_s - T_a)}{T_a} + m_c C_p \ln(\frac{T_c}{T_s})$$

**Entropy equation for the storage tank:**

$$S_s = \frac{A_c F_R}{T_s}\left[Q_s - U_L(T_s - T_a)\right] - \frac{U_s A_s (T_s - T_a)}{T_s} - \frac{m_L C_p (T_s - T_a)}{T_s} + \frac{U_s A_s (T_s - T_a)}{T_a}$$

$$+ m_c C_p \ln(\frac{T_s}{T_c}) + m_L C_p \ln(\frac{T_s}{T_L})$$

**Entropy equation for auxiliary heater:**

$$S_{aux} = m_L C_p \ln(\frac{T_D}{T_s})$$

**Entropy equation for mixing valve:**

$$S_{mix} = m_1 C_p \ln(\frac{T_D}{T_s}) + (m_L - m_1) C_p \ln(\frac{T_D}{T_L})$$

$$S_{tot} = S_c + S_s + S_{mix} + S_{aux} \tag{4.5}$$

where

$s_{mix} = 0$ if there is no mixing

$s_{aux} = 0$ if there is no auxiliary supply

Simulation of system #2 is done by solving dynamic equation of the tank together with other entropy equations for a 24-hour period using a FORTRAN program, as given in Appendix B. The integration scheme used in simulation of system#1 is utilised here also.

System#2 is simulated with the same solar radiation and ambient temperature data as system#1 and for tank volume to collector area ratios of 50,70, and 90 for total load mass flows of 50%, 100% and 150% of tank volume using each of the load profiles. Variation of the tank temperature during the hours of the day are presented in Fig.(4.32-4.34). As expected, increasing the total load mass decreases the tank temperatures for all the profiles.

Variation of entropy generation and auxiliary energy consumption for a 24-hour period are shown in Figs.(4.35-4.37) for each of the load profiles, corresponding to a constant tank volume and total load mass flow. In general entropy generation figures indicate higher values corresponding to higher load withdrawal times.

Auxiliary energy is high in all the Figs.(4.35-4.37) during early morning hours where water withdrawal is high and solar energy gain is low.

Comparison of total entropy generation and auxiliary heat requirement for Rand, ASHRAE and Vitro profiles for different tank volumes and total load mass flow rates, as listed in Table (4.3), leads to the following conclusions:

1. For a given tank size and total load mass flow, the total entropy generation and auxiliary energy requirement are higher in Vitro profile as compared to Rand and ASHRAE profiles. The main reason for the higher entropy values in Vitro profile is due to concentration of hot water demand in the early morning hours.

2. The plots of total entropy generation and auxiliary energy requirement per kg of hot water versus the total load flow for various $V_s/A_c$ ratios are shown in Figs.(4.38-4.39) for Rand profile, in Figs.(4.40-4.41) for ASHRAE profile, and in Figs.(4.42-4.43) for Vitro profile. The entropy generation Figs.(4.38, 4.40, 4.42) indicate that the minimum entropy generation condition corresponds to a total load flow approximately equal to the tank volume, irrespective of the load profile used. In general entropy generation is slighter higher for lower tank volumes.

However, the graphs of auxiliary energy requirement Figs.(4.39, 4.41, 4.43) pinpoint that for minimum auxiliary energy consumption the total heating load should be approximately equal to tank volume, and the auxiliary energy consumption is higher for higher tank volumes, regardless of the profile.

It can be concluded from the above discussion that for tank volume to collector area ratio in the range 50 to 70, the entropy generation and energy consumption will be minimum if load mass flow is approximately equal to tank size.

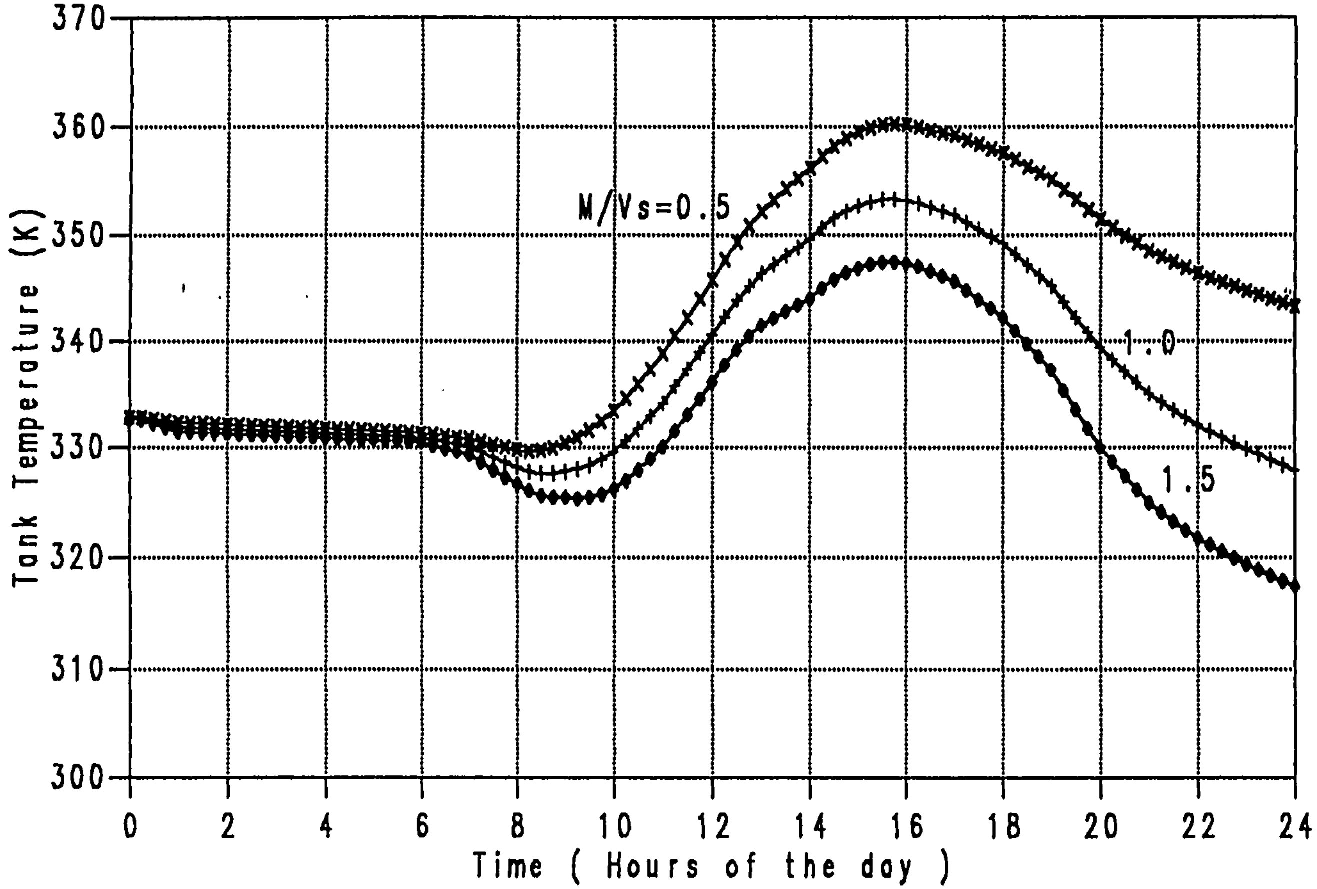



Fig. 4.32 Variation of Tank Temperature with Load
(Vs/Ac=50, Rand profile)

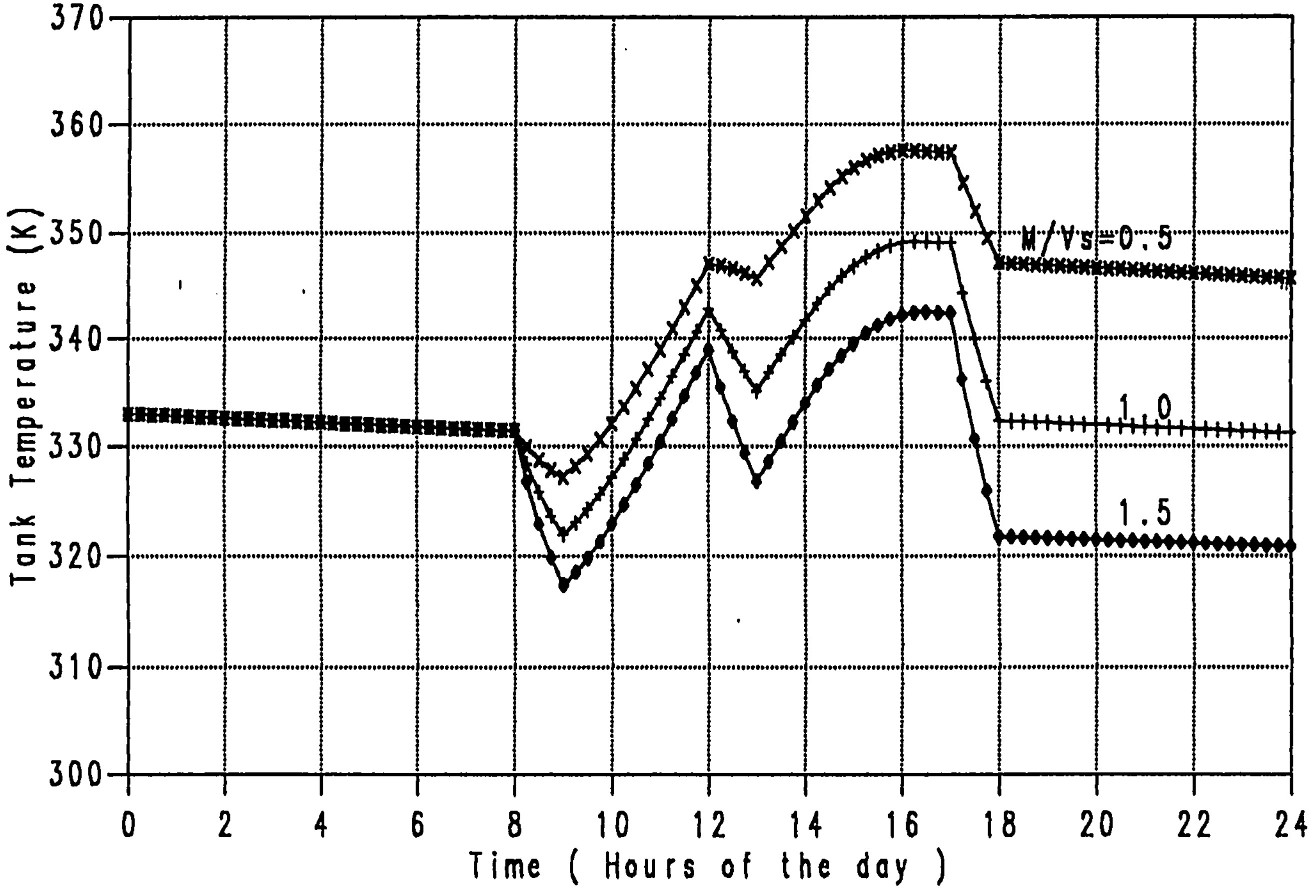



Fig. 4.33 Variation of Tank Temperature with Load
(Vs/Ac=50, ASHRAE profile)

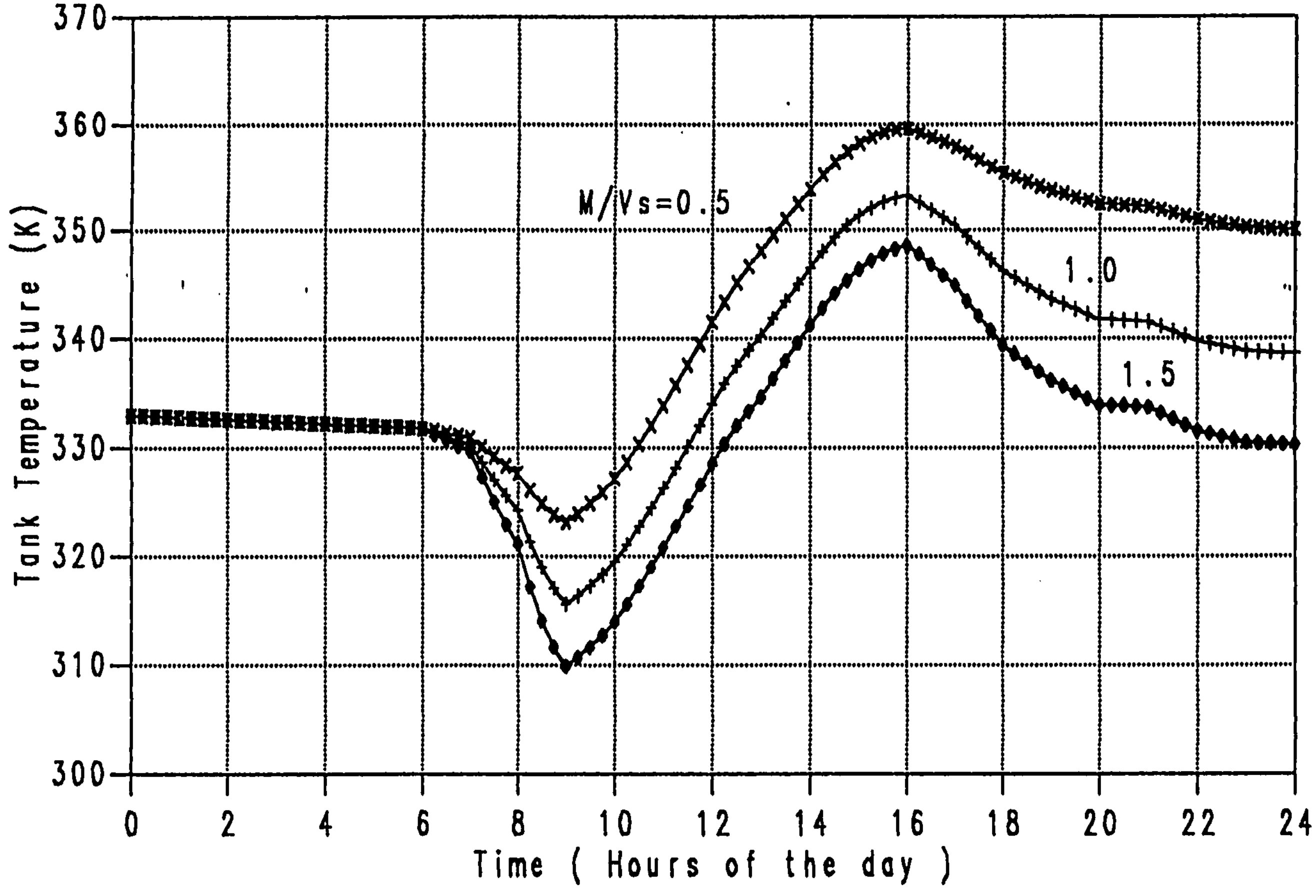



Fig. 4.34 Variation of Tank Temperature with Load
(Vs/Ac = 50, Vitro profile)

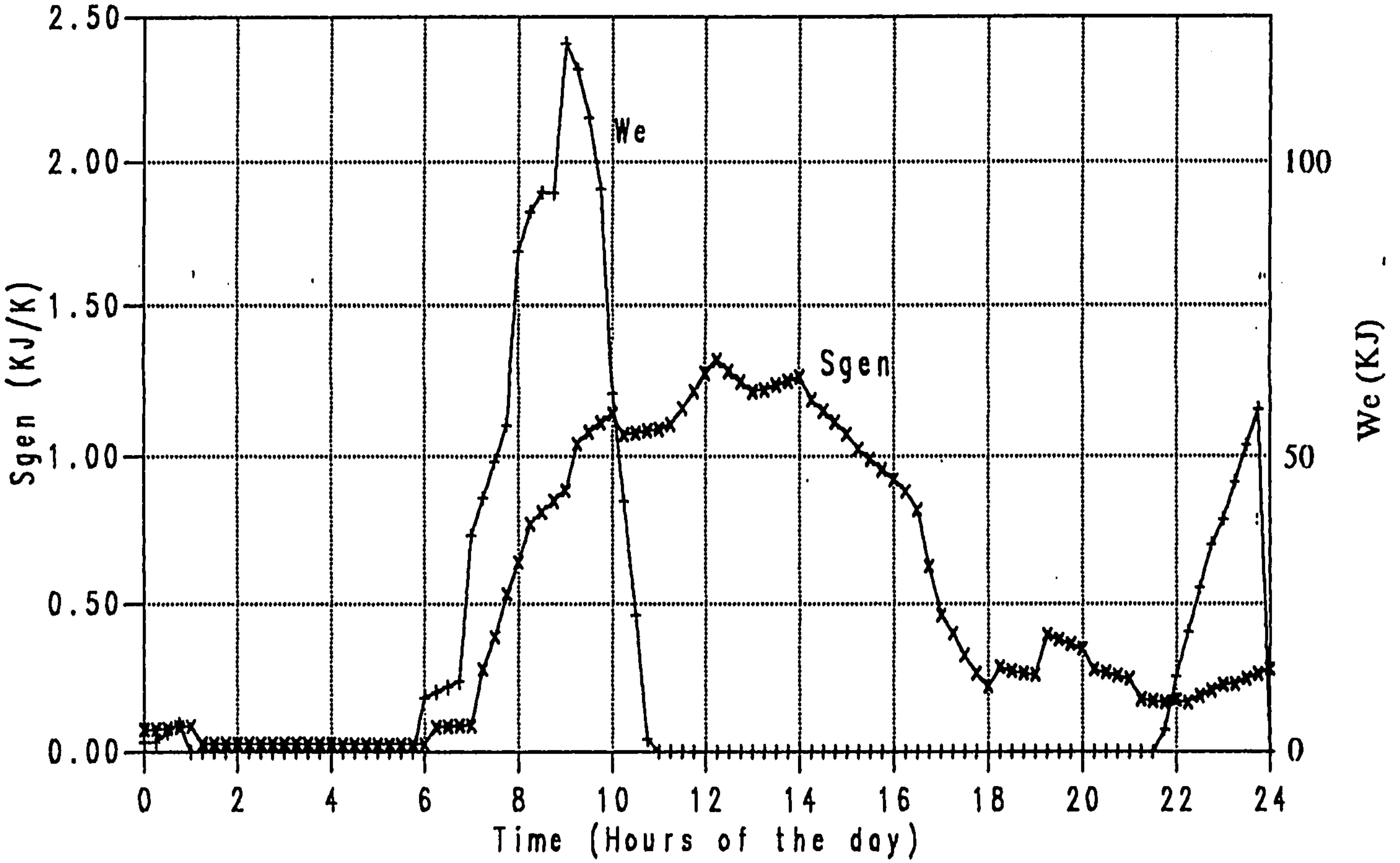



Fig. 4.35 Variation of Entropy and Auxiliary Energy
($Vs/Ac = 50$, $M/V_s = 1.0$, Rand Profile)

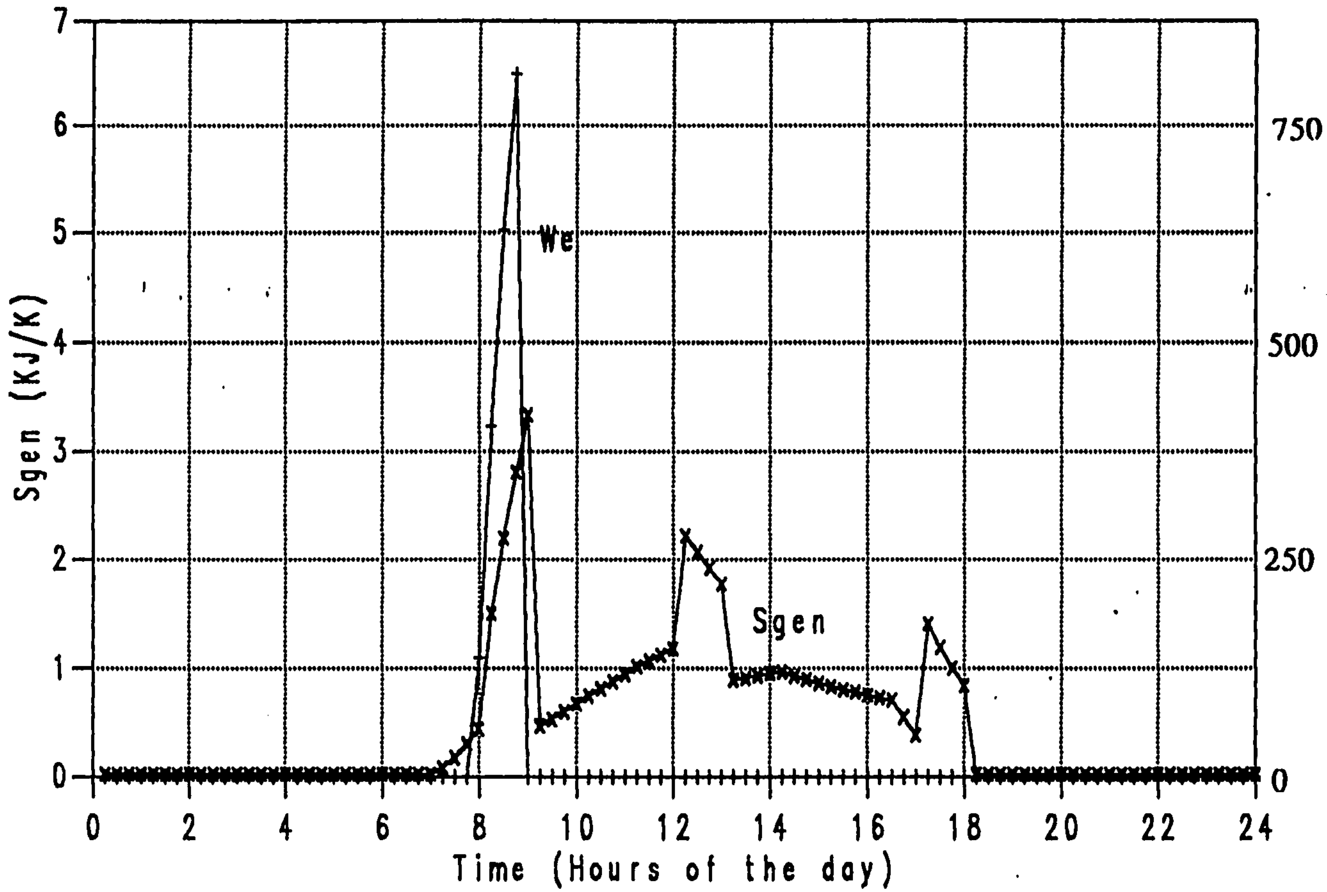



Fig. 4.36 Variation of Entropy and Auxiliary Energy (Vs/Ac=50, M/Vs=1.0, ASHRAE Profile)

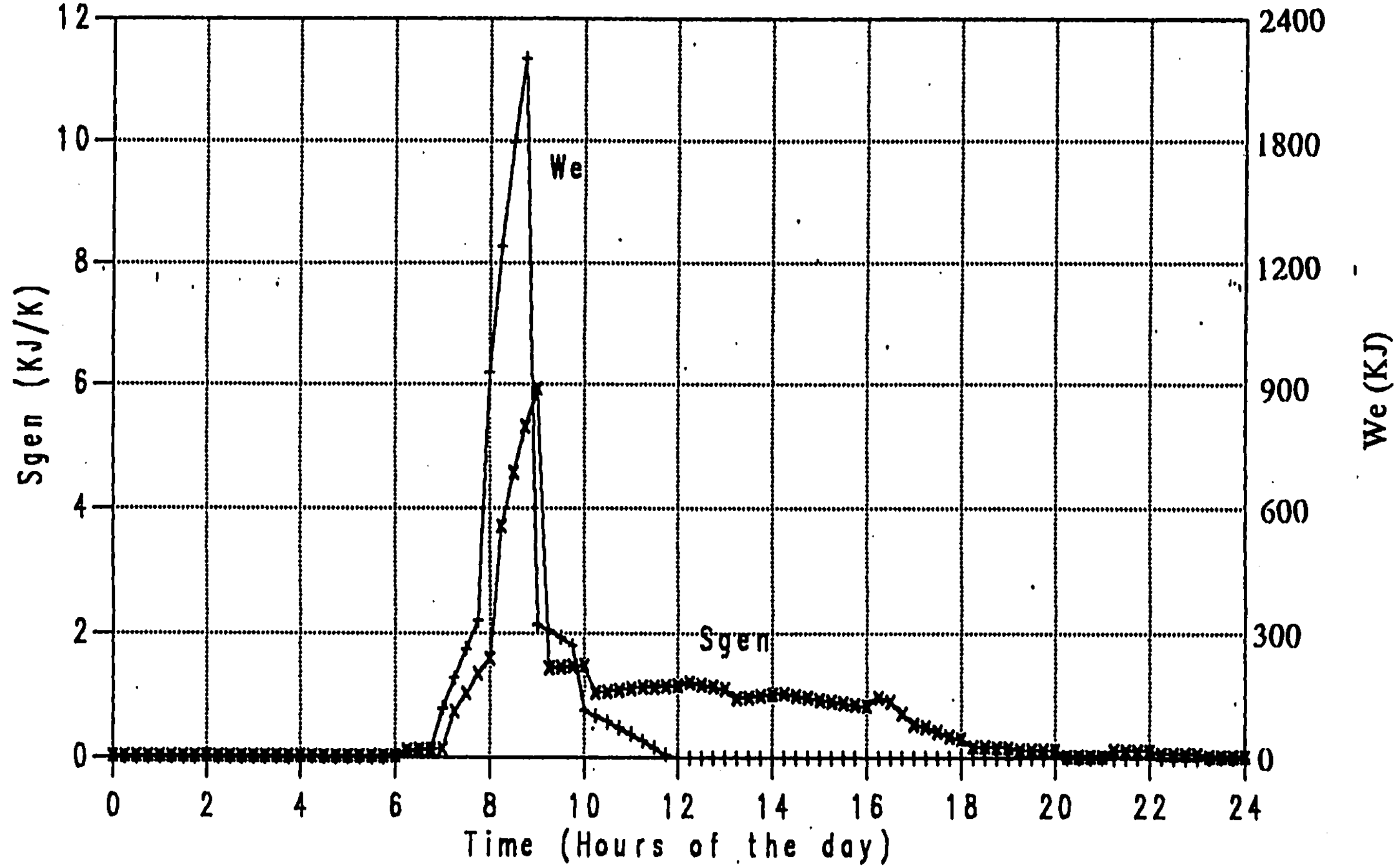



Fig. 4.37 Variation of Entropy and Auxiliary Energy (Vs/Ac = 50, M/Vs = 1.0, Vitro Profile)

TABLE (4.3) Total Auxiliary Energy Consumption and Total Entropy Generation for System#2

| | | | | Rand Profile | | Ashrae Profile | | Vitro Profile | |
|---|---|---|---|---|---|---|---|---|---|
| Case | $V_s$ (lts) | $V_s/A_c$ | $M/V_s$ | S (Total) (KJ/°K) | Aux.Heat Total(KJ) | S (Total) (KJ/°K) | Aux.Heat Total(KJ) | S (Total) (KJ/°K) | Aux. Heat Total(KJ) |
| 1 | 250 | 50 | 0.5 | 43.73 | 275.9 | 43.22 | 607.14 | 46.86 | 2262.0 |
| 2 | " | " | 1.0 | 48.40 | 1462.0 | 47.78 | 1979.2 | 63.42 | 8035.8 |
| 3 | " | " | 1.5 | 63.69 | 6721.1 | 58.23 | 5833.8 | 89.37 | 16742.0 |
| 4 | 350 | 70 | 0.5 | 44.23 | 457.8 | 43.64 | 847.3 | 48.66 | 3300.0 |
| 5 | " | " | 1.0 | 55.18 | 3981.3 | 51.72 | 3602.6 | 74.52 | 12018.0 |
| 6 | " | " | 1.5 | 86.68 | 14479.0 | 82.88 | 14107.0 | 118.40 | 26511.0 |
| 7 | 450 | 90 | 0.5 | 44.73 | 676.7 | 43.86 | 1080.8 | 57.61 | 4388.0 |
| 8 | " | " | 1.0 | 66.79 | 7732.6 | 64.34 | 7674.9 | 89.83 | 17003.0 |
| 9 | " | " | 1.5 | 118.20 | 24660.0 | 117.81 | 25359.0 | 154.77 | 38262.0 |

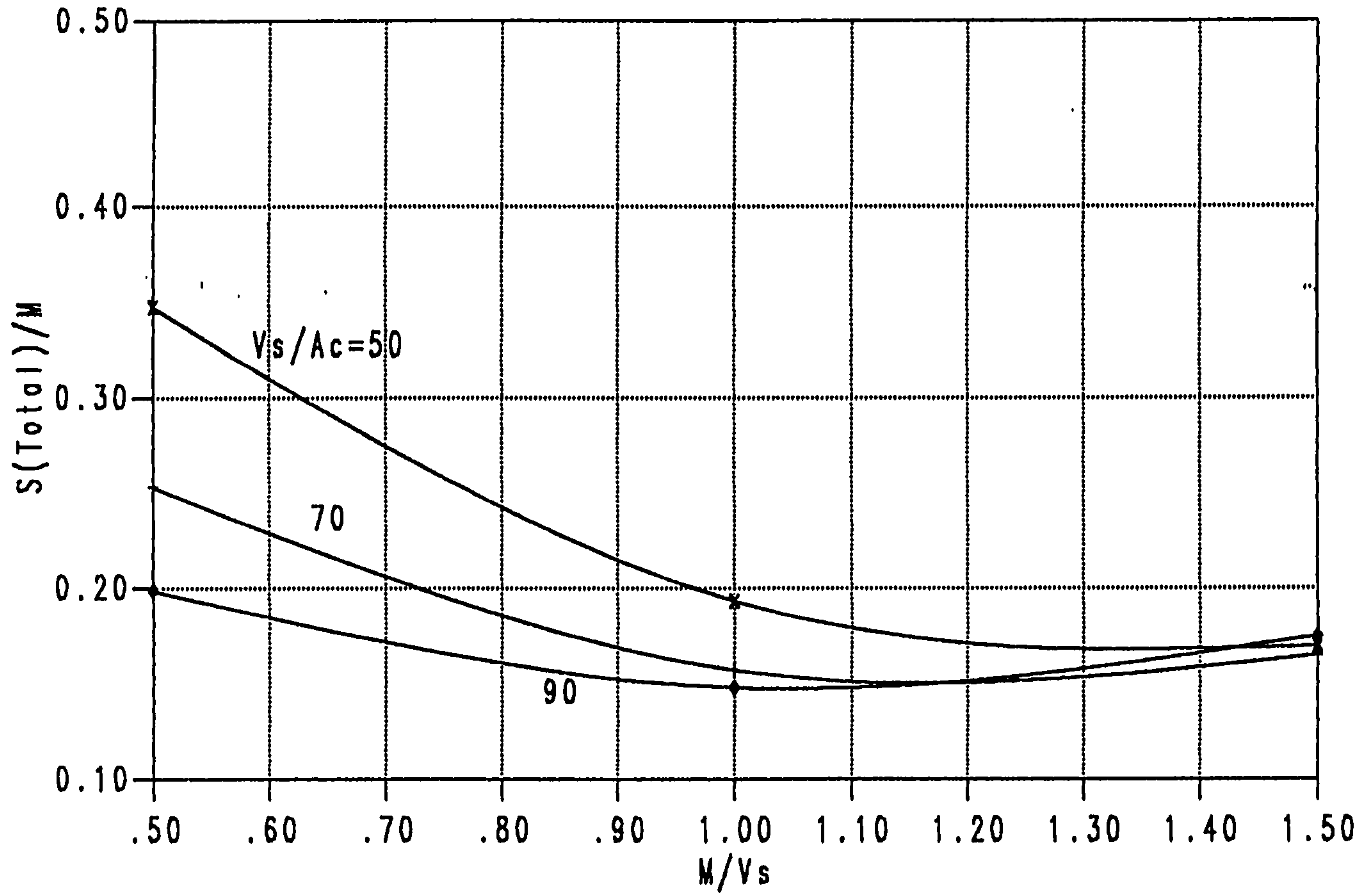



Fig. 4.38 Total Entropy generation per liter of water withdrawal (Rand profile)

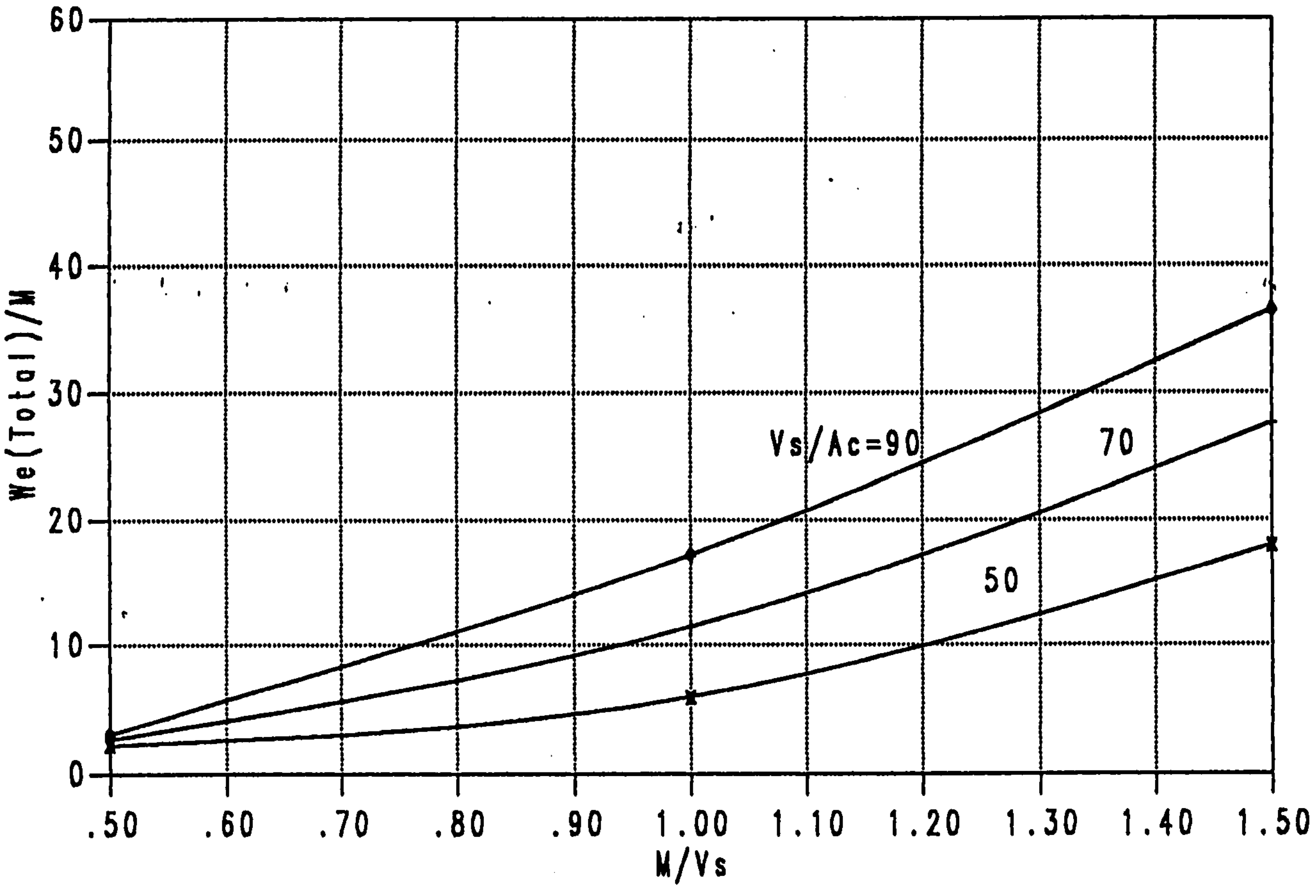



Fig. 4.39 Total Auxiliary energy per liter of water withdrawal (Rand profile)

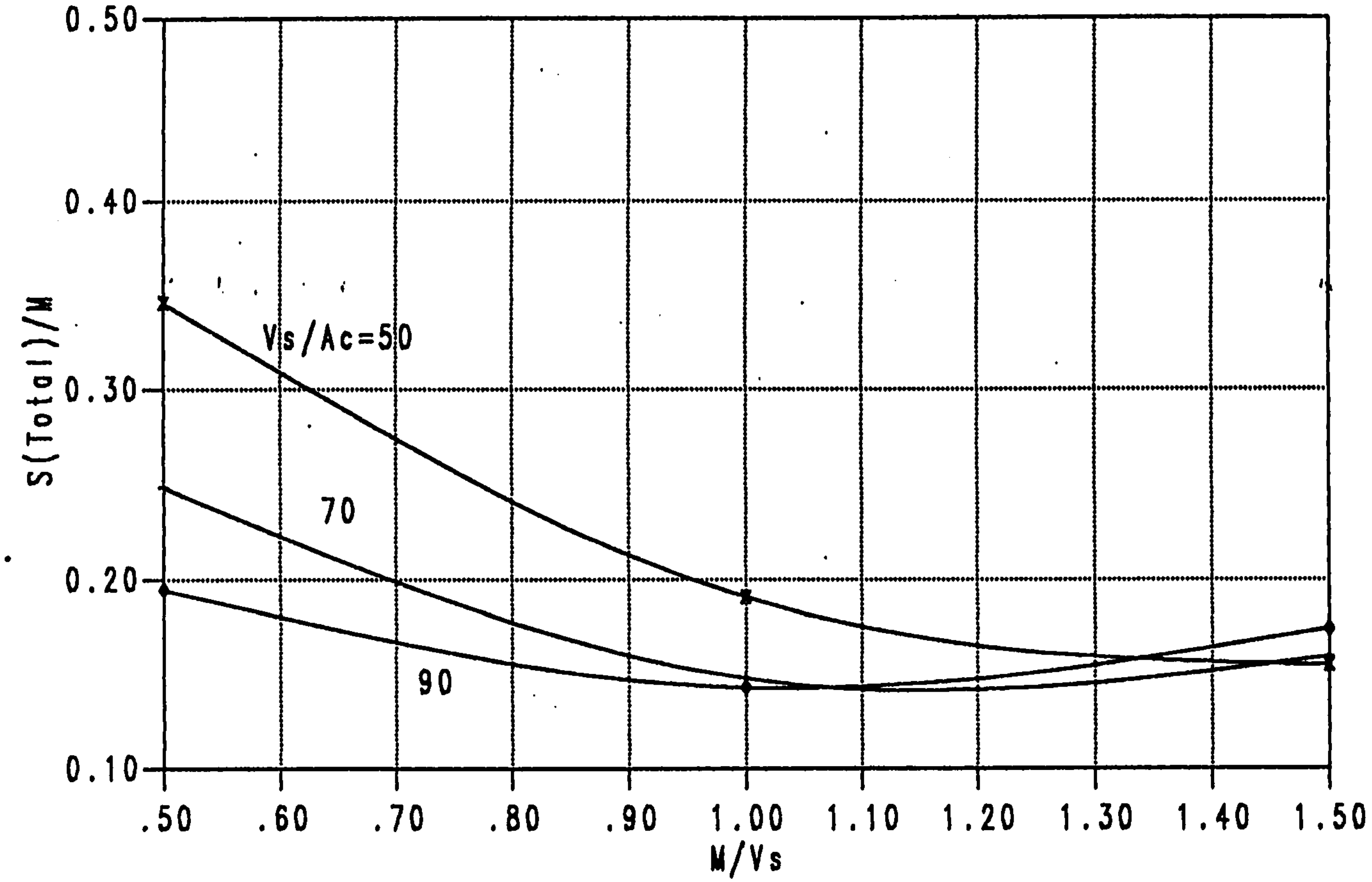



Fig. 4.40 Total Entropy generation per liter of water withdrawal (ASHRAE profile)

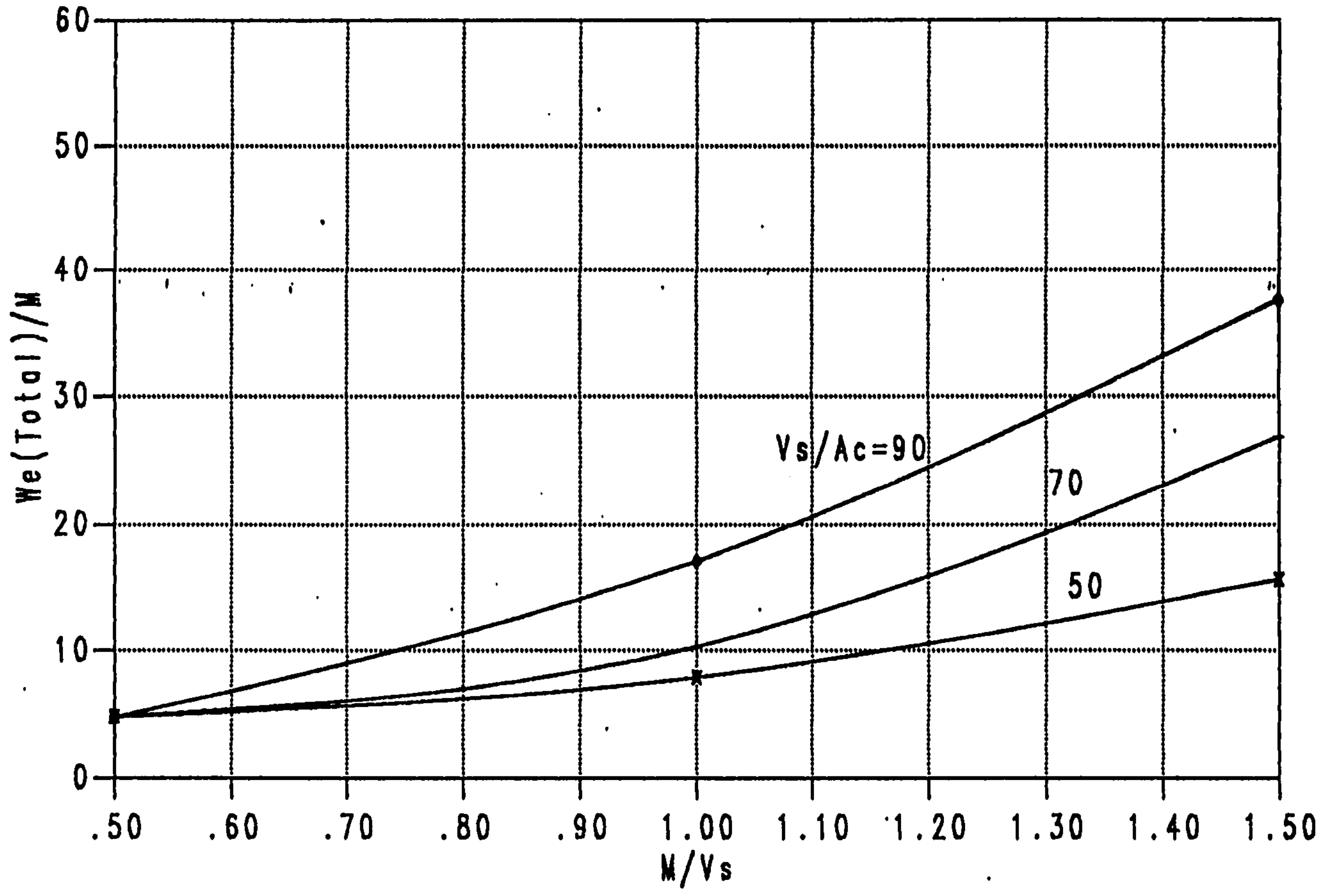



Fig. 4.41 Total Auxiliary energy per liter of water withdrawal (ASHRAE profile)

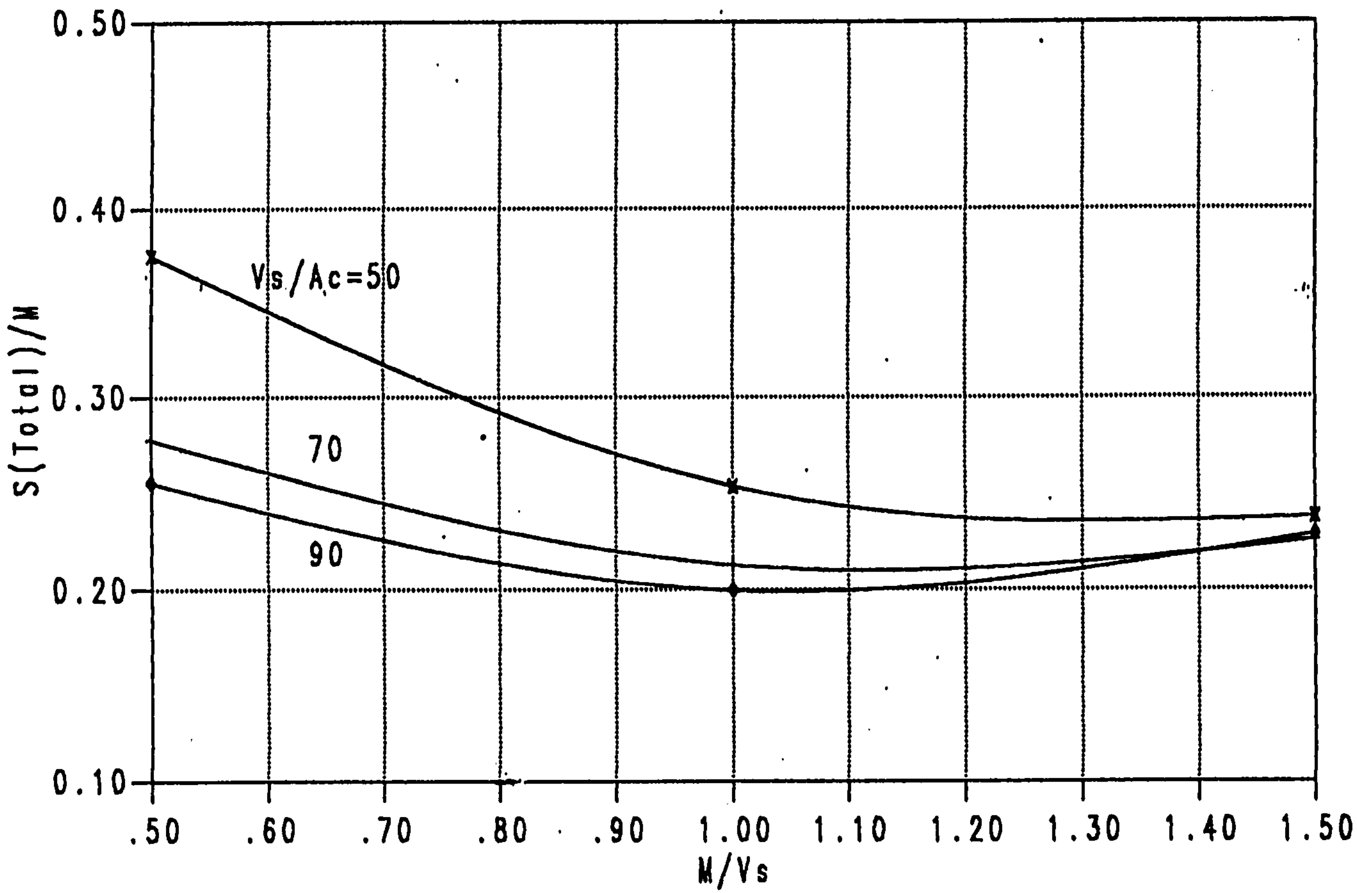



Fig. 4.42 Total Entropy generation per liter of water withdrawal (Vitro profile)

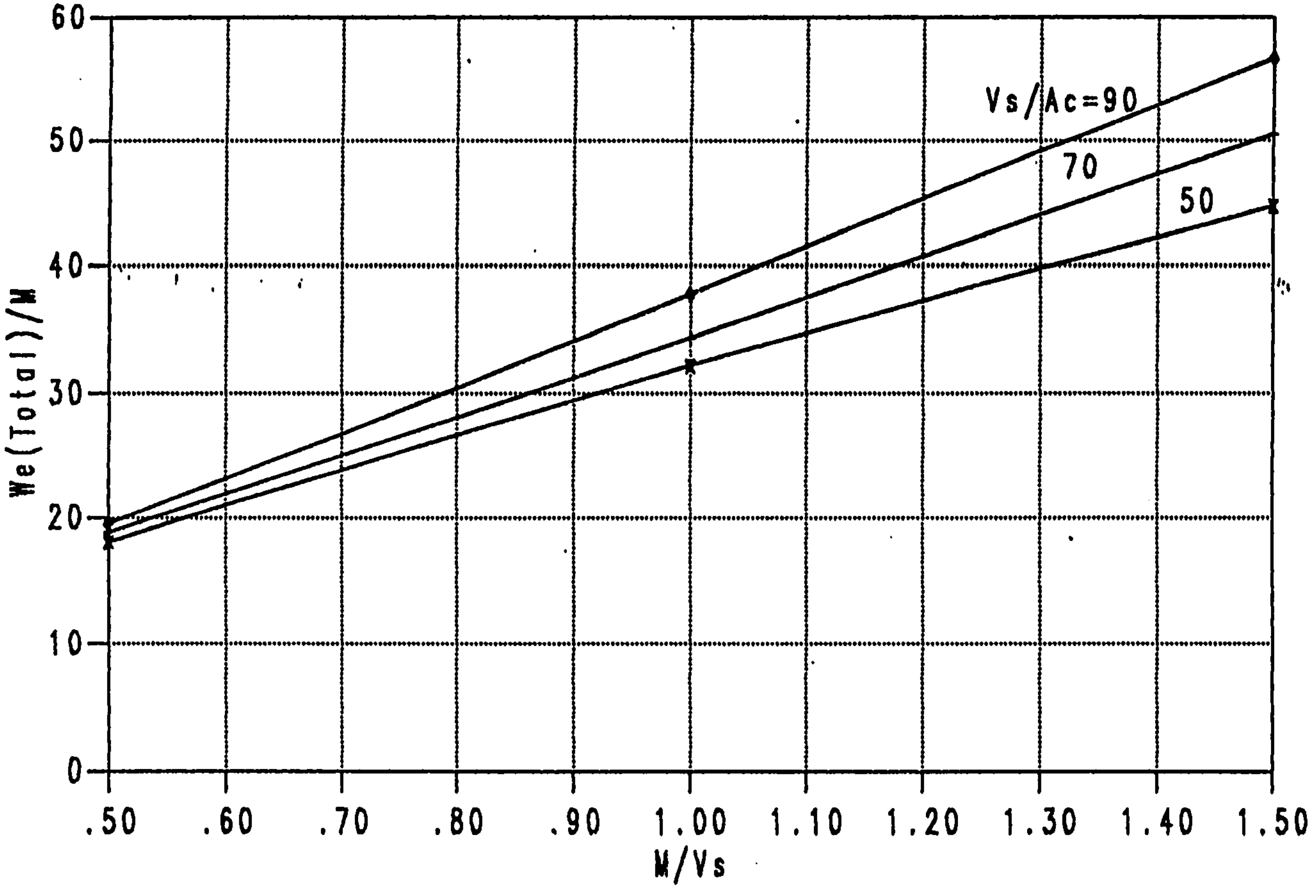



Fig. 4.43 Total Entropy generation per liter of water withdrawal (Vitro profile)

**4.4 Comparison of the two hot water heating systems:**

Comparison of total entropy generated and total auxiliary energy consumption of the two systems for Rand, ASHRAE and Vitro profiles for different tank volumes and total load mass rates as listed in Tables (4.2-4.3) leads to the following conclusions:

1. For a given profile, tank size and heating load, the entropy generation in the system#2 is considerably less, as compared to the system#1.

2. For a given profile, tank size and heating load, there is a significant saving in auxiliary energy consumption in system#2 as compared to system#1. This finding is in good agreement with the earlier works [26]. The concrete reason for the dramatic reduction in auxiliary energy consumption in system#2 as compared to system#1 can be attributed to the fact that the system#2 maximizes the use of solar collector output and avoids unnecessary heating of water in the storage tank as in system #1.

3. For system#1, Rand profile gives minimum entropy generation and minimum auxiliary energy consumption as compared to ASHRAE and Vitro profiles, where as in system#2 Rand and ASHRAE profiles generate less entropy and consume less energy as compared to Vitro profile. In system#2 Rand and ASHRAE profile results show similar behaviour with regard to entropy generation and auxiliary energy required.

4. For system#1, $M/V_s$ ratio of 1 and $V_s/A_c$ ratio of 50 provides the minimum entropy generation and auxiliary energy consumption per liter of water. However, for system#2, $M/V_s$ ratio of approximately and $V_s/A_c$ ratio of 90

provides the minimum entropy generation per liter of water, but for minimum auxiliary energy consumption $V_s/A_c$ must be 50 and $M/V_s$ must be less than or equal to 1.0. 5. Provision of auxiliary heater in series with the storage system results in minimum total entropy generation and minimum total auxiliary energy consumption.

# CHAPTER 5

# OPTIMIZATION STUDIES

## 5.1 Introduction:

This chapter addresses in sufficient depth the application of optimal control theory to entropy minimization problem. Optimal control theory is playing an increasingly important role in the design of modern systems. It is a new and direct approach to the synthesis of complex systems and has been made feasible by the development of the digital computer. The objective of optimal control strategy is to determine the control signals that will cause a process to satisfy the physical constraints and at the same time minimize (or maximize) some performance criterion, which evaluates the performance of the system quantitatively. In a broad sense performance measure is a translation of the system's physical requirement into mathematical terms. Selection of performance measure is a subjective matter. In all that follows in the current research, performance of the system is evaluated by a measure of the entropy generation. Once the performance measure for the system has been decided, the next task is to determine a control function that minimizes this criterion. Two methods of accomplishing the minimization are the minimum principle of Pontryagin and the method of dynamic programming developed by R.E. Bellman. The variational approach of Pontryagin leads to a nonlinear two-point boundary value problem (TPBVP) that must be solved to obtain an optimal control Law. Dynamic programming leads to a functional equation that is amenable to solution by use of a digital computer. Dynamic programming is indeed a direct method in that it attempts to directly minimize the given cost function without obtaining a two-point boundary value problem.

### 5.2 Optimal Control Problem for System#1:

Objective in the optimal control of the mentioned SDHW systems, is to obtain an optimum load mass flow profile while minimizing the total entropy generated during a 24-hour period and keeping the total withdrawn mass equal to the corresponding case in simulation. During optimization initial and final tank temperatures will also be kept equal to the corresponding simulation case. Thus the optimization problem is to find the distribution of load mass flow rate $m_L$ while minimizing

$$S_{TOTAL} = \int_0^T S_{gen}(T_s, m_L, W_e, V_s, p, t)\, dt \tag{5.1}$$

where instantaneous entropy generation in the system $S_{gen}$ is a function of tank temperature, $T_s$, load mass flow rate $m_L$, auxiliary energy $W_e$, storage tank volume $V_s$, time t and other parameters p which are either kept constant, such as collector area, main water temperature, etc or variations of those parameters are independent of the optimization process, such as solar insolation and ambient temperature.

This optimization is subjected to the total mass flow rate constrained as:

$$M = \int_0^T m_L\, dt = \text{constant} \tag{5.2}$$

which indicates that the total load mass flow will be kept constant during optimization. It is also subjected to the dynamic equation of the storage tank.

$$\frac{dT_s}{dt} = f(T_s, T_c, W_e, V_s, p, t) \tag{5.3}$$

where $T_s(0) = T_{s0}$, $\qquad T_s(T) = T_{st}$

are kept equal to the simulation results, and the static equation of the collector is given as:

$$Q_u = A_c F_R \left[ I_T(\tau\alpha) - U_L(T_s - T_a) \right] = m_c C_p (T_c - T_s) \tag{5.4}$$

Eq.(5.4) is used to solve $T_c$ in terms of $T_s$ and other parameters. $T_c$ is substituted in Eq.(5.3) to obtain it in the form of

$$\frac{dT_s}{dt} = f(T_s, m_L, W_e, V_s, p, t) \tag{5.5}$$

Optimization is to be performed such that as long as there is a load flow from the tank, the temperature of the tank should be equal or above the desired temperature $T_D$. If it is above (due to solar gain), certain amount of water from tank will be mixed with water from mains to provide the desired water temperature at the optimum flow rate. If tank temperature is below the desired water temperature, a certain amount of auxiliary energy will be added to bring it to the desired temperature $T_D$ for the optimum mass flow rate.

Optimizations like simulations are also performed using a constant collector flow rate of 0.02 kg/(m$^2$sec) as long as the useful energy of the collector $Q_u$ is positive. Using the optimum flow rate as given by Eq.(2.12)

$$m_{opt} = \left[\frac{A_c^2 U_L (F')^2 [S - U_L(T_{in} - T_a)](1 - \frac{T_a}{T_{in}})}{6C_f C_p}\right]^{1/4}$$

did not affect the results appreciably. So, for the sake of simplicity collector flow rate is kept constant at the above mentioned value. During optimizations some values of $V_s/A_c$, $M/V_s$ ratios are used for parametric studies, as it is done in simulation studies and the results are presented in this chapter.

Thus the optimization problem will be reduced to determining load mass flow rate $m_L$ by minimizing :

$$S_{TOTAL} = \int_0^T S_{gen}(T_s, m_L, W_e, p, t)\, dt \tag{5.6}$$

subjected to

$$\frac{dT_s}{dt} = f(T_s, m_L, T_c, W_e, p, t) \tag{5.7}$$

where $T_s(0) = T_{so}$, $\quad T_s(T) = T_{st}$

and

$$M = \int_0^T m_L\, dt \qquad (m_L \geq 0) \tag{5.8}$$

and the other constraints are as explained in detail above.

## 5.3 Optimal Control Problem for System#2

Objective of the control problem for system#2 is the same as the one for system#1. The difference is in system configuration and the corresponding equations. Similar equations as Eq.(5.1) and (5.2) are used in this part. Eq.(5.3) is modified because auxiliary energy is supplied into the load mass flow rate directly as shown in Fig.(2.2). Optimization is performed such that as long as the tank temperature is above the desired temperature (due to solar gain), water from the tank is mixed with water from mains to provide desired temperature $T_D$. If the tank temperature is below the desired water temperature, water is withdrawn from tank and is heated by addition of auxiliary energy.

During optimization the collector mass flow rate is kept constant $0.02\ kg/(m^2 sec)$ and the same ratios of $V_s/A_c$ and $M/V_s$ are used for parametric studies as used in simulations of these systems with different load profiles. Initial and final tank temperatures are kept equal to the corresponding temperature obtained from simulation for the same system and for the same $V_s/A_c$ and $M/V_s$ value.

## 5.4 Solution Methods for Optimization Problem:

Due to the existing nonlinearity in entropy equations of systems and nonlinear constraints imposed on systems, solution of the optimization problem present a lot difficulties to obtain convergence. The different optimization methods that are studied and applied to the existing problem are reviewed below as the way they are applied to the current problem using the Eqns.(5.6 - 5.8).

### 5.4.1 Solution Using Pontryagin's Maximum Principle with Kalaba's Method [53]

The integral constraint in Eq.(5.8) can be modified as

$$\int_0^T (m_L - \frac{M}{T})\, dt = 0 \tag{5.9}$$

this can be combined with Eq.(5.6) and the optimization problem can be solved by minimizing the integral

$$J_1 = \int_0^T [S_{gen} + q(m_L - \frac{M}{T})]dt \tag{5.10}$$

where q is the lagrange multiplier and is equal to a unknown constant. It is a trial and error procedure to determine the magnitude of the constant lagrange multiplier q which minimizes the integral constraint Eq.(5.8). The solution of Eq.(5.10) is unchanged if expressed in the form

$$J_2 = \int_0^T [S_{gen} + qm_L]dt - q(T)M \tag{5.11}$$

Now the value of the lagrange multiplier q can be obtained by adjoining the contraint

$$\dot{q} = 0 \tag{5.12}$$

with unknown initial condition. Here the parameter q is treated as a state variable. Then with dynamic constraint in Eq.(5.7), the optimization problem requires the minimization of

$$J = \int_0^T [S_{gen} + qm_L + \lambda_1 f + \lambda_2 .0]dt - q(T)M \tag{5.13}$$

where $\lambda_1$ and $\lambda_2$ are Lagrange multipliers. Use of Pontryagin's Maximum Principle leads to the following two-point boundary-value equations and associated boundary conditions as

$$\lambda_1 + \frac{\partial H}{\partial T_s} = 0\,, \qquad T_s(0) = T_{so}\,, \qquad T_s(T) = T_{st}$$

$$\lambda_2 - \frac{\partial H}{\partial q} = 0\,, \qquad \lambda_2(0) = 0\,, \quad \lambda_2(T) + M = 0 \tag{5.14}$$

$$\frac{\partial H}{\partial m_L} = 0$$

where H is the Hamiltonian defined as

$$H = S_{gen} + qm_L + \lambda_1 f + \lambda_2 .0 \tag{5.15}$$

Solution of this problem via Newton-Raphson method, as outlined in Ref.[53], could not be obtained due to the following reasons:

1. Variation of Hamiltonian in Eq.(5.14) is singular due to the control variable (load mass flow rate $m_L$). In other words, $\frac{\partial H}{\partial m_L} = 0$ does not provide an equation as a function of $m_L$. It required an initial assumption and iteration procedure.

(2) Two-point boundary value equations in Eq.(5.14) are highly nonlinear. Furthermore, working of the system is not continuous, depending on variables, there are jumps to different trajectories. Solutions of these equations have been

attempted via shooting method and finite difference method using IMSL subroutines available at Data Processing Centre (DPC) of KFUPM. Convergence could not be obtained due to the above mentioned reasons and iterations are found to be highly sensitive to initial conditions.

**5.4.2 Solution using Gradient Method [54]:**

Integral constriant in Eq.(5.8) is converted to a state equation as

$$\frac{dy}{dt} = m_L \,, \quad y(0) = 0, \quad y(T) = M \tag{5.16}$$

with the above equation, the optimization problem is to minimize

$$S_{TOTAL} = \int_0^T S_{gen}(T_s, m_L, W_e, p, t)\, dt \tag{5.17}$$

subjected to

$$\frac{dT_s}{dt} = f(T_s, T_c, W_e, p, t) \tag{5.18}$$

where $T_s(0) = T_{so}, \qquad T_s(T) = T_{st}$

and

$$\frac{dy}{dt} = m_L \quad (m_L \geq 0)\,, \quad y(0) = 0, \quad y(T) = M$$

and the other constraints are as explained in section 5.2.

Defining the Hamiltonian as

$$H = S_{gen} + \lambda_1 f + \lambda_2 m_L \tag{5.19}$$

and using Pontryagin's Maximum Principle the following two-point boundary equations and associated boundary values are obtained

$$\dot{\lambda}_1 + \frac{\partial H}{\partial T_s} = 0 \qquad T_s(0) = T_{so} \qquad T_s(T) = T_{st}$$

$$\dot{\lambda}_2 - \frac{\partial H}{\partial y} = 0 \tag{5.20}$$

$$\frac{dT_s}{dt} = f(T_s, T_c, W_e, p, t)$$

$$\frac{dy}{dt} = m_L \qquad y(0) = 0, \qquad y(T) = M$$

$$\frac{\partial H}{\partial m_L} = 0$$

In theory, 4 equations can be solved using 4 boundary conditions. Unfortunately, $\frac{\partial H}{\partial m_L} = 0$ does not yield a relation for $m_L$, but only provides a relation between $\lambda_1$ and $\lambda_2$. For this reason, the solution of this problem is an iterative process. It requires guessing a load profile, solving the equations by guessing lagrange multipliers $\lambda_1$ and $\lambda_2$ by using the relationship between $\lambda_1$ and $\lambda_2$. Then $\frac{\partial H}{\partial m_L}$ is determined and the variation in load profile is calculated as

$$\Delta m_L = -K \frac{\partial H}{\partial m_L} \tag{5.21}$$

where K is a convergence coefficient. A new profile is calculated as

$$m_L^{new} = m_L^{old} + \Delta m_L \tag{5.22}$$

and process is repeated until $\Delta m_L^{new}$ or $\Delta J^{new}$ varies little, where $\Delta J^{new}$ is defined as

$$\Delta J^{new} = \int_0^T \frac{\partial H}{\partial m_L} \Delta m_L \, dt = -K \int_0^T \left[\frac{\partial H}{\partial m_L}\right]^2 dt \tag{5.23}$$

After the convergence, the new value of $\lambda_1$ or $\lambda_2$ must be guessed until the required boundary conditions are satisfied.

Due to the aforementioned reasons, this method also did not give convergence.

### 5.4.3 Solution using Dynamic Programming Method [55]

Since the two earlier methods did not give converging results, dynamic programming has been used to solve the optimization problem. It was not attempted first, because of it's large storage requirements and long execution time. However Dynamic programming when confronted with Pontryagin's maximum principle has a couple of advantages which led to its application in the present investigation. It can accomodate highly nonlinear system equations and performance criteria; constraints of extremely general nature can be easily handled; it determines global optimum within quantization accuracy; it is inherently simple in comprehension and implementation; another and perhaps very important characteristic is related to the fact that solutions are obtained for an entire family of problems. This occurs because we obtain the minimum cost and optimum decisions at every admissible quantized state and stage; one more consequence of this approach is that feedback control or decision policy solution is obtained.

**5.5 Entropy minimization for system#1 via Dynamic Programming:**

The equations for dynamic programming solution of system#1 are the same as Eqns.(5.17-5.18).

Following the dynamic programming algorithm explained in Appendix C, optimization proceeds as follows:

1. State variables $T_s$ and y are quantized with uniform increments. Each quantized value of $T_s$ is indexed with j1 where j1 = 1,2,.......J1, also each quantized value of y is indexed with j2 where j2 = 1,2........J2. After discretization the state eqautions take the following forms:

$$T_{sn} = T_{so} + \frac{\Delta t}{M_s C_s}\Big\{A_c F_R[Q_{sol} - U_L(T_s - T_a)] - A_s U_s(T_s - T_a)$$

$$- m_l C_p(T_s - T_a) + W_e\Big\} \tag{5.24}$$

$$y_n = y_o + \Delta t \times m_L$$

where

$T_{so}$ value of $T_s$ at the beginning of time step

$T_{sn}$ value of $T_s$ at the end of time step

$y_o$ Total load mass flow at the beginning of time step

$y_n$ Total load mass flow at the end of time step

with boundary conditions:

$$T_o(0) = 333°K$$

$$T_s(T) = \text{constant determined by simulation results}$$

$$y(0) = 0$$

$$y(T) = M = \text{Total mass flow determined simulation result}$$

2. Decision variables $m_L$ and $W_e$ are quantized with uniform increments. Each discrete value of $m_L$ is indexed as $m_1 = 1,2,........M1$ and each discrete value of $W_e$ is indexed as $m_2 = 1,2,........M2$

3. The stage variable is time and is quantized with a step of 900 seconds. Total number of stages are 97. Each stage is indexed as k where $k=1,2,......K$

4. The performance criteria Eq.(5.17) which we seek to minimize is discretized as:

$$I = \sum_{k=1}^{K}\left\{-\frac{Q_{sol}}{A_c} + \frac{A_c U_L(T_s - T_a)}{T_a} + m_c C_p \ln\left(\frac{T_c}{T_s}\right)\right.$$

$$- A_s U_s \frac{(T_s - T_a)}{T_s} - m_L C_p \frac{(T_s - T_L)}{T_s} + \frac{W_e}{T_s} + A_s U_s \frac{(T_s - T_a)}{T_a}$$

$$+ m_L C_p \ln(\frac{T_s}{T_l}) \Big\} \times \Delta t$$

$$= \sum_{k=1}^{K} L(m_1, m_2) \Delta t \tag{5.25}$$

where $L(m_1,m_2)$ is the entropy generated at a given stage k, taking the values of variables at the beginning of stage.

5. Boundary conditions for a particular case are obtained from simulation results for the same system and for the same case. A zero value for entropy (minimum cost function) is taken at the states corresponding to boundary conditions at stage K=97. The inadmissible states are assigned large values to handle them numerically. These costs serve as initial conditions in initiating the backward optimization process.

The complete state space or grid of quantized states for optimization is shown in Fig.(5.1)

6. An iterative functional equation is defined to evaluate the optimal decisions as:

$$I(j1,j2,k\text{-}1) = L(m_1,m_2) + I(k) \tag{5.26}$$

where $L(m_1,m_2)$ is the entropy generated in the current stage (k-1) with decisions $(m_1,m_2)$ and $I(k)$ is the total entropy at stage k.

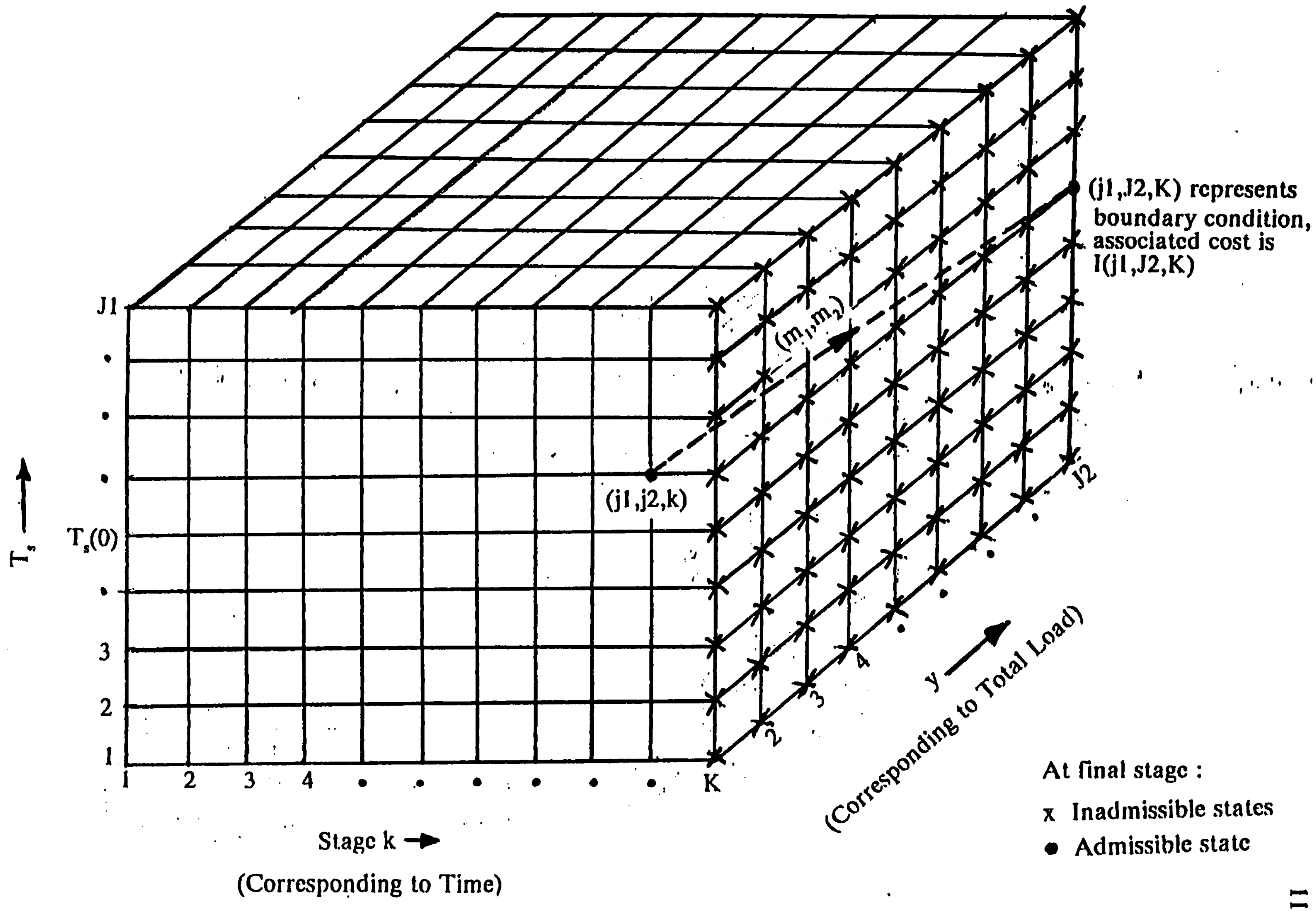



Fig. 5.1 Optimization space for Dynamic Programming

7. A quantized state (j1,j2) at stage (k-1) is considered. At this state, all of the admissible decisions $m_L(m1)$ and $W_e(m2)$ are applied. For each of these decisions the entropies at the current stage $L(m_1,m_2)$ are determined using the current $T_s$ and y values.

8. For each of $m_L(m1)$ and $W_e(m2)$ decisions, the next states $T_{sm}$, $y_n$ at stage k are determined from the governing differential Eq.(5.24).

9. A check is made whether the states $T_{sm}$, $y_n$ are within the discretized range. If they are outside the range no further consideration is given to these states and in future they are identified by assigning large entropy values.

10. If the next states $T_{sm}$, $y_n$ fall within the discretized range, a linear interpolation scheme is used to compute the entropy I(k) at these points.

The next states are determined from discrete values when

$$T_s(j1, j2) < T_{sm} < T_s(j1+1, j2) \text{ and}$$

$$y(j1, j2) < y < y(j1, j2+1)$$

Then a two dimensional linear interpolation scheme is used to determine I(k) values as shown in Fig.(5.2) and Eq.(5.27)

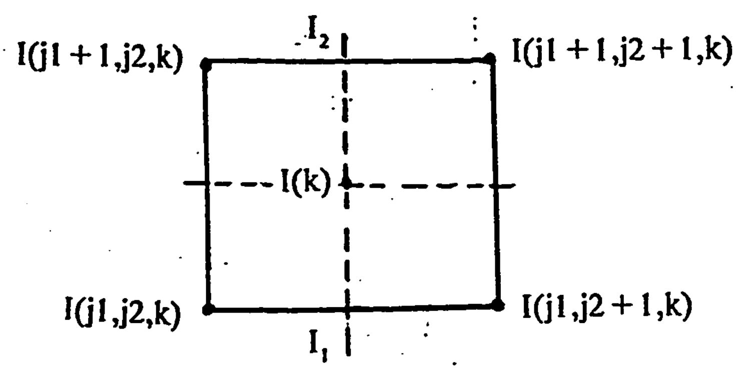



Fig. 5.2 Two dimensional interpolation

The interpolation is first done along y or j2 direction,

$$I1 = I(j1,j2,k) + \frac{I(j1,j2+1,k) - I(j1,j2,k)}{y(j2+1) - y(j2)}(y_n - y(j2))$$

$$I2 = I(j1+1,j2,k) + \frac{I(j1+1,j2+1,k) - I(j1+1,j2,k)}{y(j2+1) - y(j2)}(y_n - y(j2))$$

The interpolation is then done along $T_s$ or j1 direction

$$I(k) = I(1) + (I2 - I1)\frac{T_{sn} - T_s(j1)}{T_s(j1+1) - T_s(j1)} \tag{5.27}$$

11. The total entropy generated at state ($T_s$, y) in stage (k-1) is given by Eq.(5.26). This is the quantity which is to be minimized by choice of $m_L(m1)$ and $W_e(m2)$. Minimization is achieved by comparing all the quantities computed. The minimum value will be the minimum cost at state (j1,j2) at stage (k-1). The optimal decisions at this state $m_L(j1,j2,k-1)$, $W_e(j1,j2,k-1)$ correspond to the minimum entropy at the state indexed by I(j1,j2,k-1)

12. The procedure is repeated at every quantized state at stage (k-1). When this has been done the minimum entropy generated and the optimal decisions $m_L(j1,j2,k-1)$, $W_e(j1,j2,k-1)$ are known for all quantized states at stage (k-1). The procedure is continued until k=1 is reached.

I(j1,j2,k), $m_L(j1,j2,k)$, $W_e(j1,j2,k)$ constitute the dynamic programming solution to the problem.

**Following procedure is adopted to trace a optimal trajectory :**

In this context optimal trajectory refers to optimum load mass flow profile during a 24-hour period, between fixed initial and final tank temperatures while minimizing the total entropy generated.

For a given initial condition ($T_s(1)$, $y(1)$). The first decisions in the sequence are evaluated as

$$m_L(1) = m_L(T_s(1), y(1), 1)$$

$$W_e(1) = W_e(T_s(1), y(1), 1)$$

The next states along the sequence are then found by applying the system difference Eqns.(5.24). These states may or may not be quantized states. If they are, then next decisions in the optimum sequence are evaluated as

$$m_L(2) = m_L(T_s(2), y(2), 2)$$

$$W_e(2) = W_e(T_s(2), y(2), 2)$$

If they are not quantized states, then the values of $u_L(2)$ and $W_e(2)$ must be determined by interpolation scheme as illustrated in Fig.(5.1) and Eq.(5.27). The recovery procedure continues untill the complete decision sequences $m_L(1)$,......$m_L(K)$, $W_e(1)$,......$W_e(K)$, and optimal trajectories $T_s(1)$,......$T_s(K)$, $y(1)$,......$y(K)$ have been obtained.

Euler's integration scheme was utilized to solve the governing differential Eqns.(5.24). A FORTRAN program, listed in Appendix D, was developed to implement the dynamic programming algorithm. The program reads, as input, the known parameter values, the initial values of entropy at the boundary conditions at final stage K, the history of solar insolation and ambient temperature as given in Fig.(4.1). The program prints out, at every admissible quantized state and stage, the minimum entropy generated, minimum auxiliary energy consumed and the associated optimal flow rate. Also for given initial conditions, the program traces the optimal path - between the initial and final stage.

It is clear from simulation studies in Chapter 4 that a system subjected to Rand profile generates less entropy and consumes less auxiliary energy. Therefore to compare the Rand profile results with optimum load profile results, several optimization runs are made for the same cases (Tank volume to collector area $V_s/A_c$, and total load mass flow to tank volume $M/V_s$ ratios) for which simulation has already been done using Rand profile. For each optimization run, the boundary conditions are taken from the corresponding Rand profile simulation results. Optimization program satisfies all the boundary conditions, this is an evidence of convergence criterion of dynamic programming.

Optimum load profiles for $V_s/A_c = 50$ and $M/V_s$ ratios of 0.5, 1.0, 1.5 are shown in Figures. (5.3-5.5)

Optimum load profiles shown in Figures (5.3-5.5) have similar patterns and show that for minimum entropy generation associated with auxiliary energy consumption, the hot water demand should be concentrated in afternoon hours. This matches fairly well with earlier works [43, 56].

Variation of tank temperature over a 24-hour period, for tank volume to

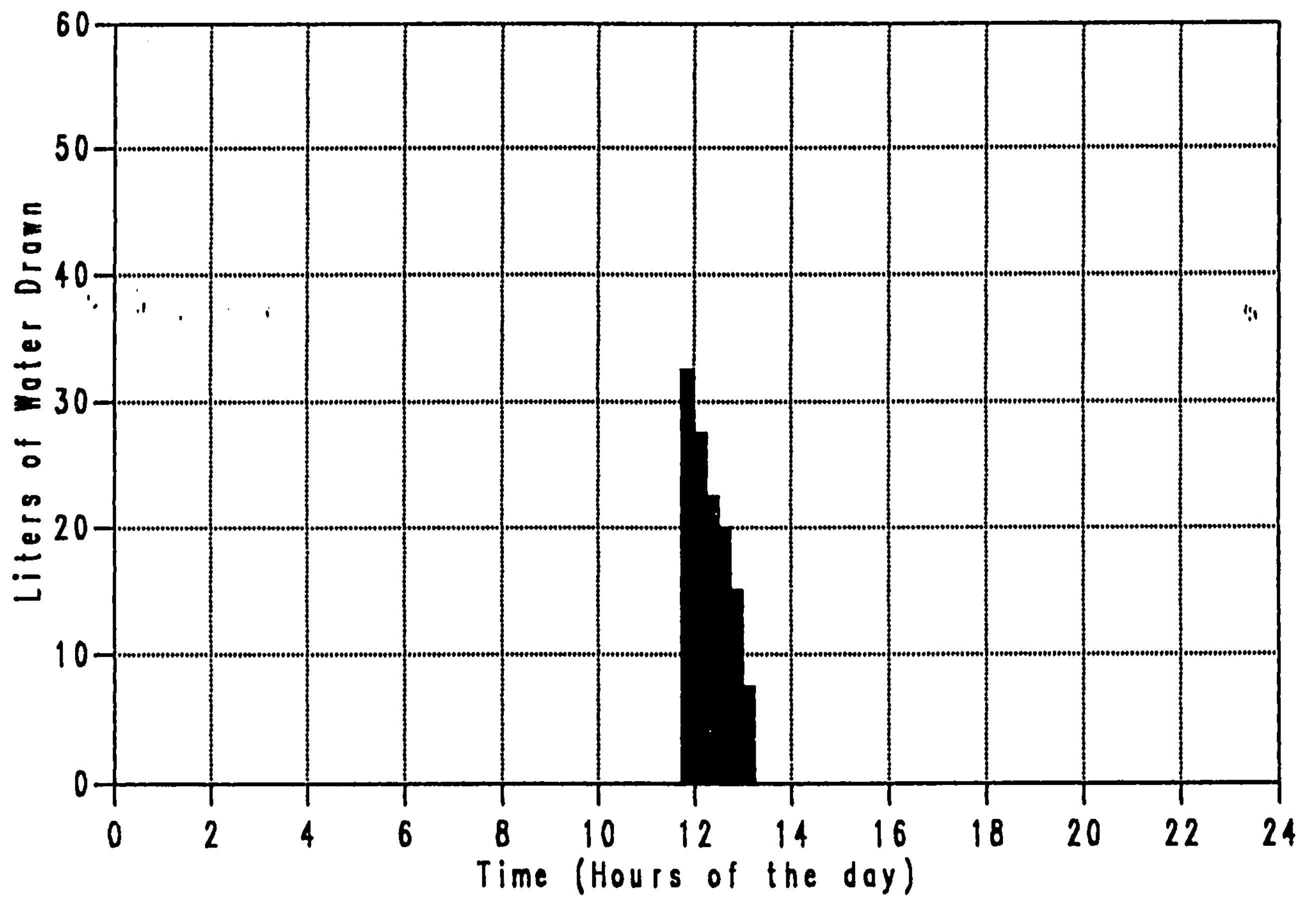



Fig. 5.3 Optimum load profile (System#1, Vs/Ac = 50, M/Vs = 0.5)

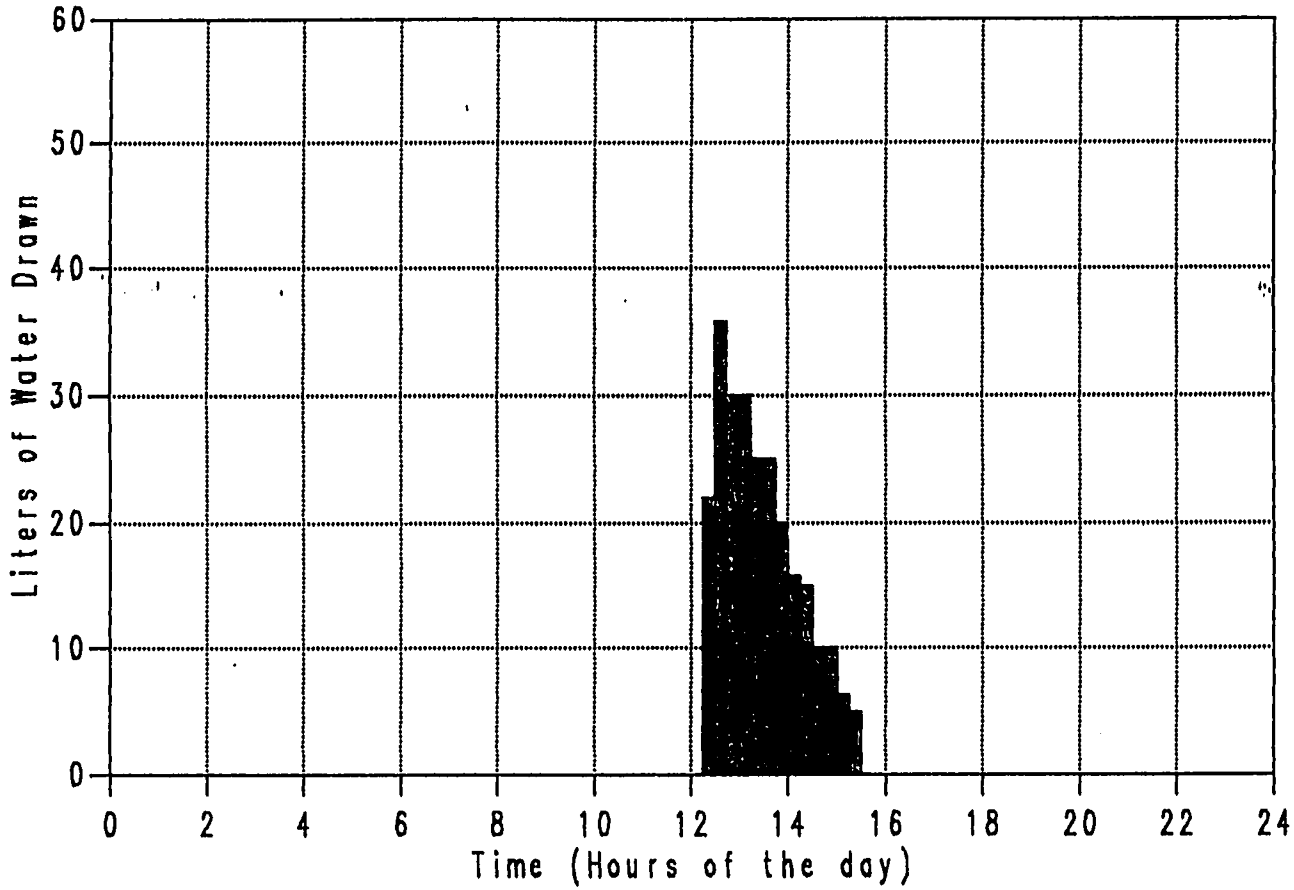



Fig. 5.4 Optimum load profile (System#1, Vs/Ac = 50, M/Vs = 1.0)

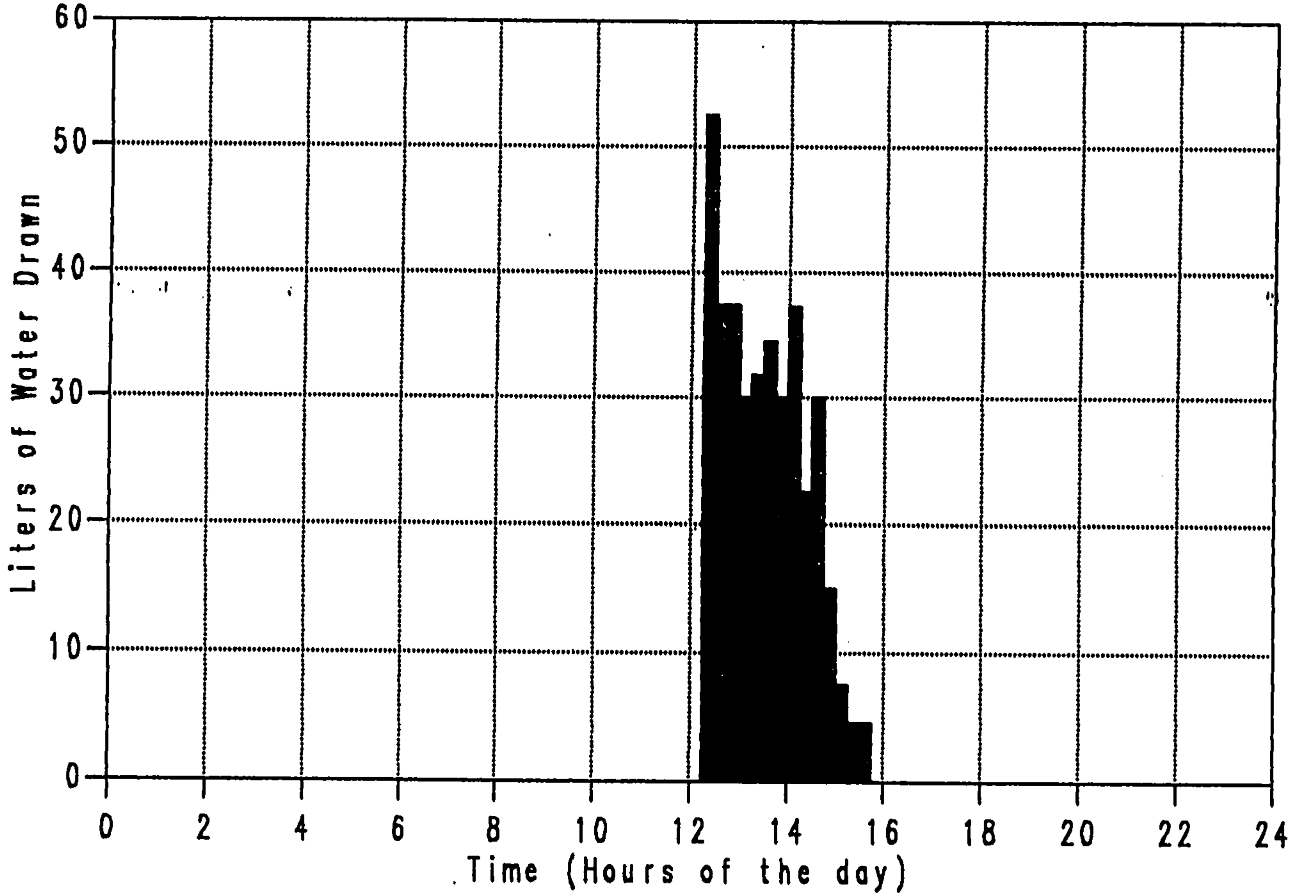



Fig. 5.5 Optimum load profile (System#1, Vs/Ac = 50, M/Vs = 1.5)

collector ratios of 50, 70, 90 and total load to storage volume ratios of 0.5, 1.0, 1.5 are displayed graphically in Fig.(5.6-5.8). As in simulation studies, drop in temperature occurs with increase in hot water heating load, and temperature increases with solar insolation.

Variation of entropy generation and auxiliary energy over a period of 24 hours for $V_s/A_c$ = 50 and for different $m/V_s$ ratios of 0.5, 1.0, 1.5 are shown in Figures (5.9-5.11). Entropy generation and auxiliary energy required are higher at higher load withdrawal times.

Comparison of entropy generation and auxiliary energy requirement for different ratios of $V_s/A_c$ and $M/V_s$ is shown in Table (5.1). For a given tank size increase in heating load results in increase in entropy generation and auxiliary energy consumption. The increase is more for bigger sizes.

The plots of total entropy generation and auxiliary energy requirement per kg of hot water versus total load flow for different $V_s/A_c$ ratios are shown in Figures (5.12-5.13). Graph in Fig.(5.12) suggests that $V_s/A_c = 50$ and, $M/V_s = 1.0$ is optimum from the point of view of minimum entropy generation. For larger tank sizes entropy generation tends towards the minimum for lower amount of water withdrawal. Fig.(5.13) indicates that for minimum auxiliary energy consumption, heating load corresponds to approximately $V_s/A_c = 50$ and, $M/V_s = 1.0$, but minimum point is not so sensitive to $M/V_s$ ratio as in the entropy generation. For larger tank sizes auxiliary energy consumption tends towards the minimum for lower amount of water withdrawal.

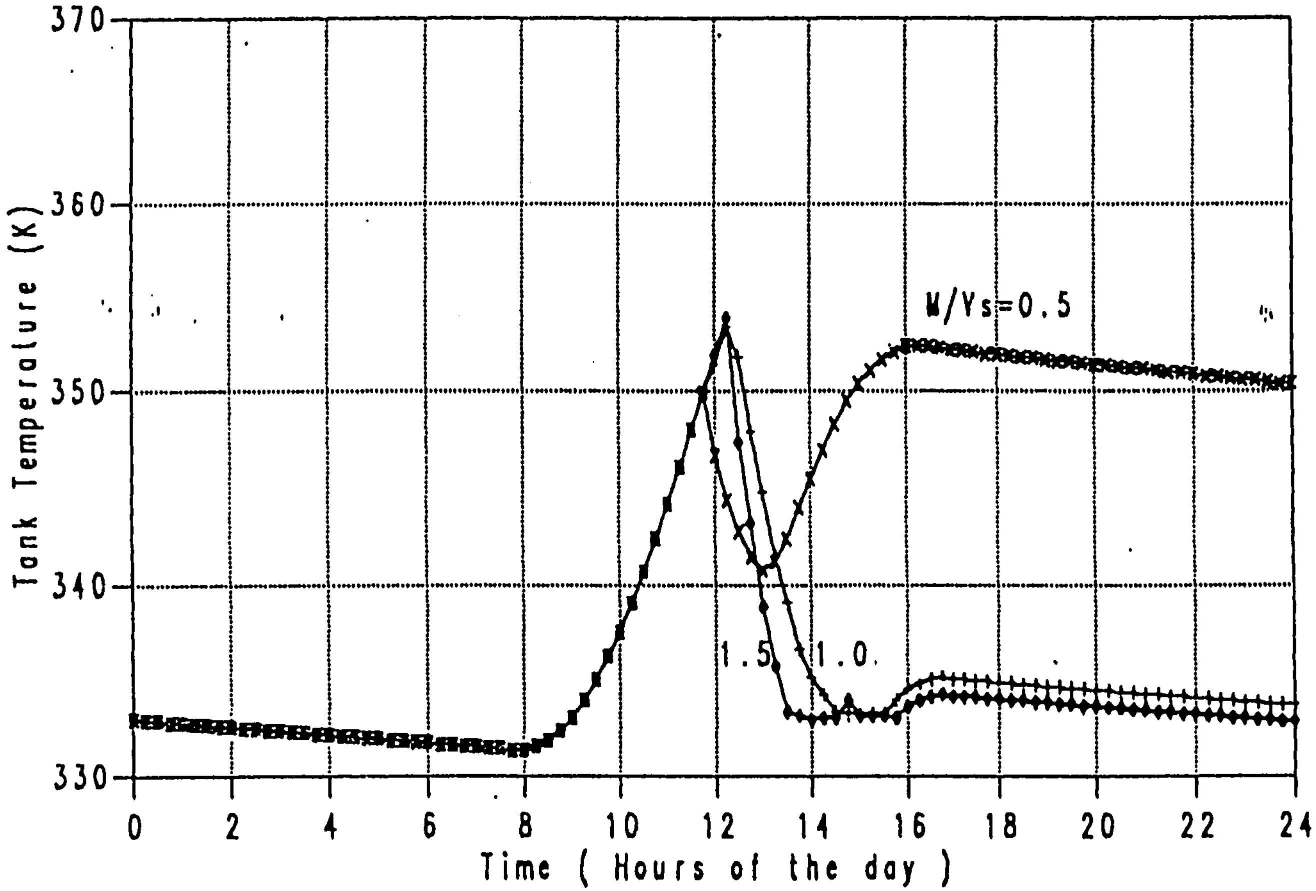



Fig. 5.6 Variation of Tank Temperature with Load
(Vs/Ac = 50, Optimum load profile)

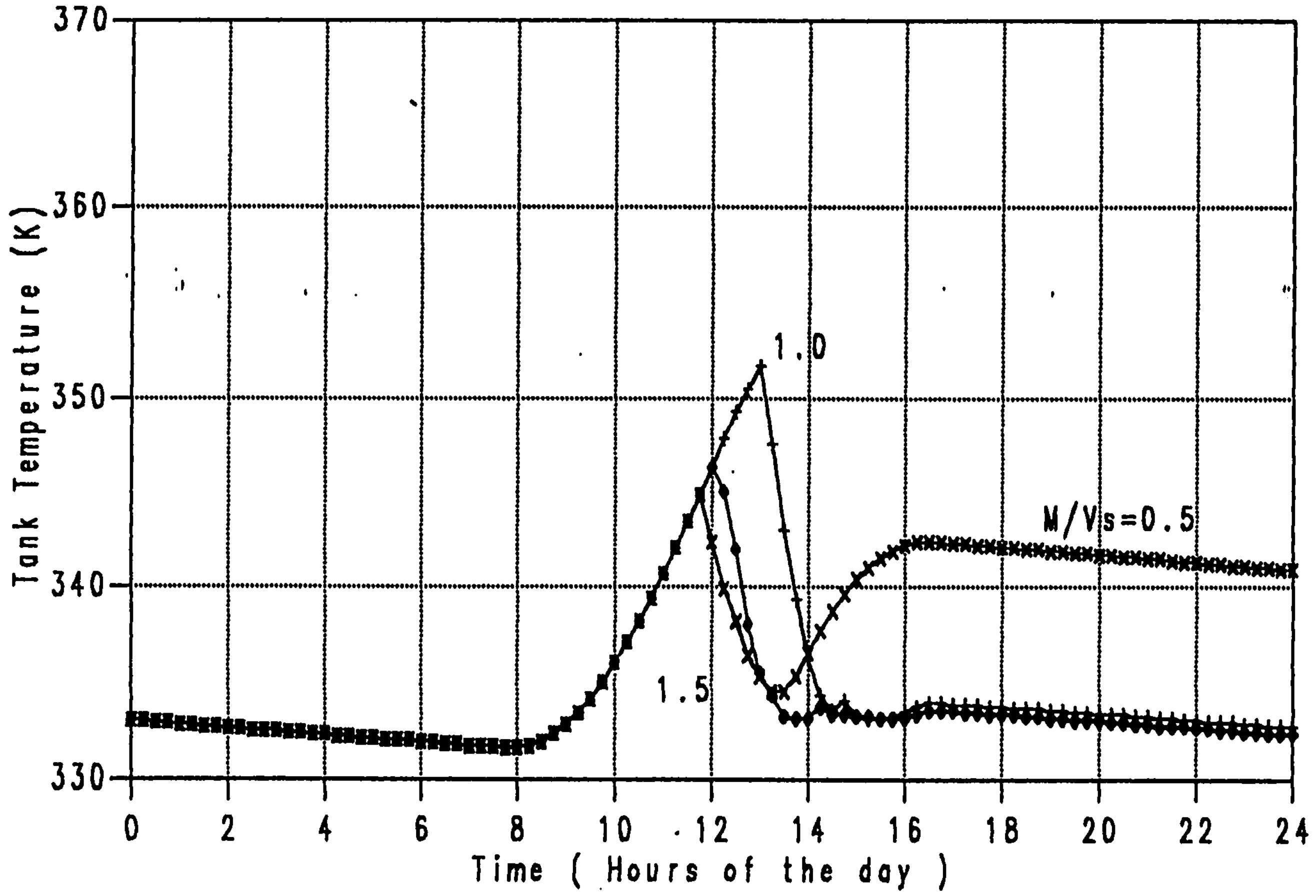



Fig. 5.7 Variation of Tank Temperature with Load
(Vs/Ac=70, Optimum load profile)

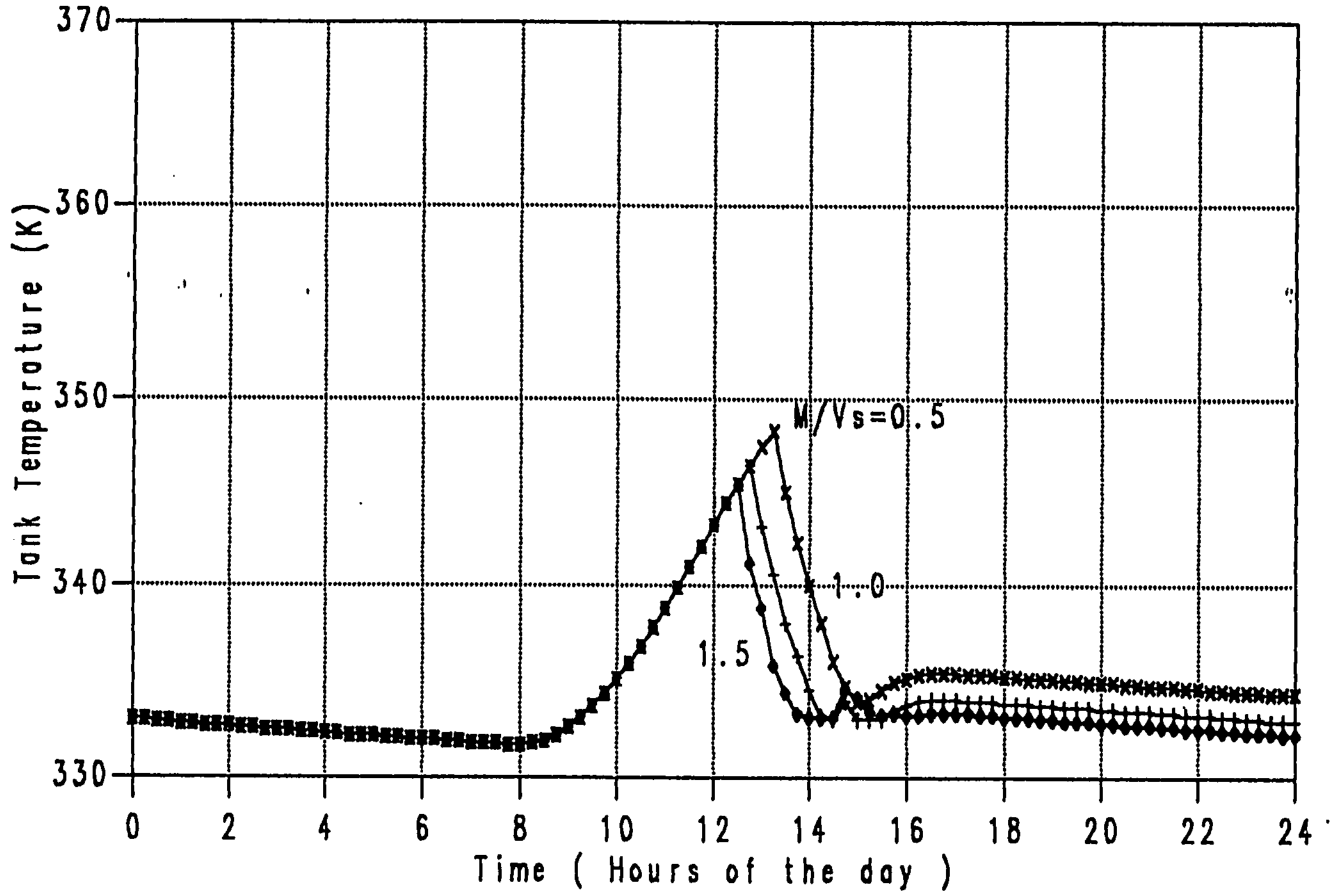



Fig. 5.8 Variation of Tank Temperature with Load (Vs/Ac=90, Optimum load profile)

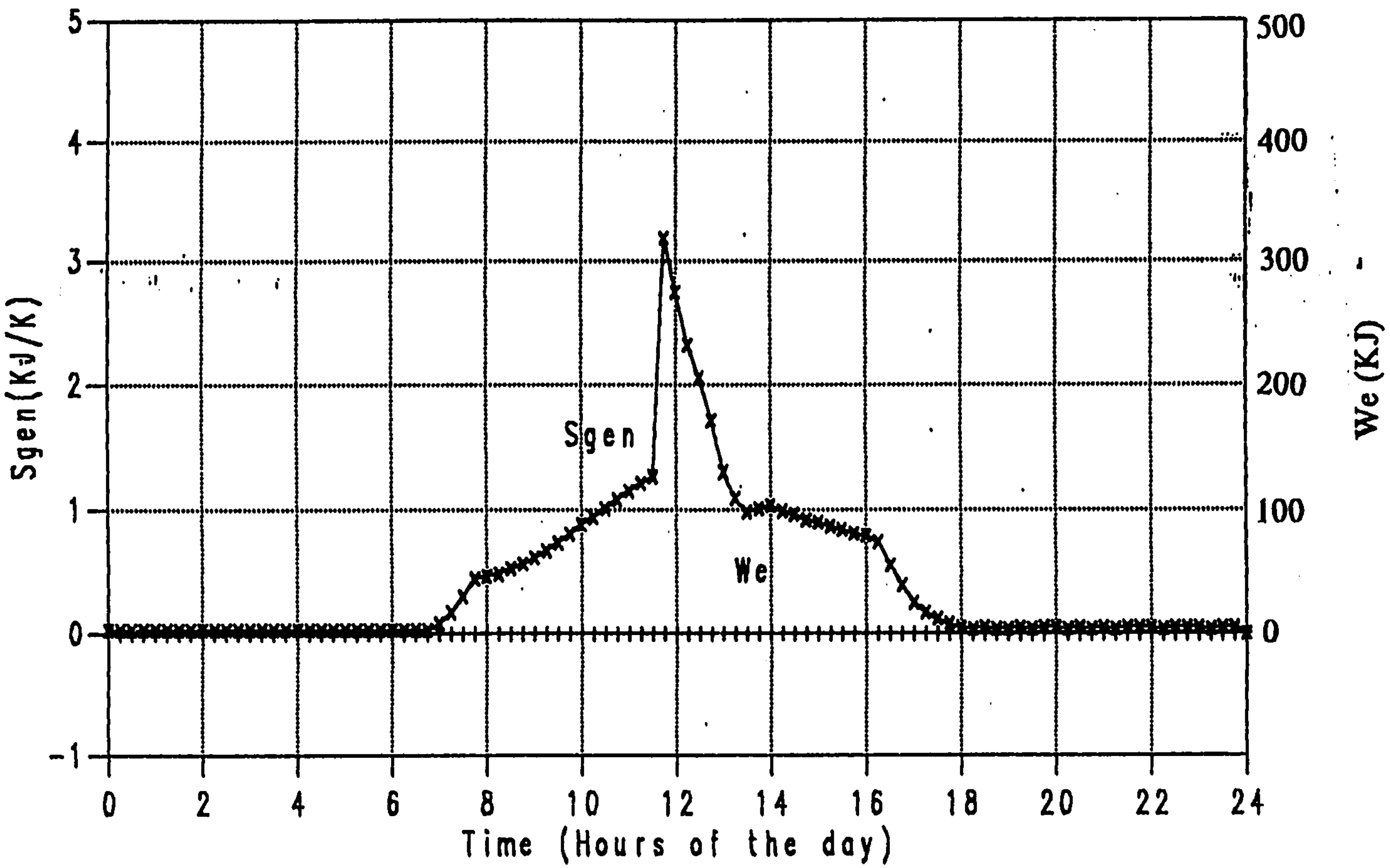



Fig. 5.9 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 50, M/Vs = 0.5, Optimum load Profile)

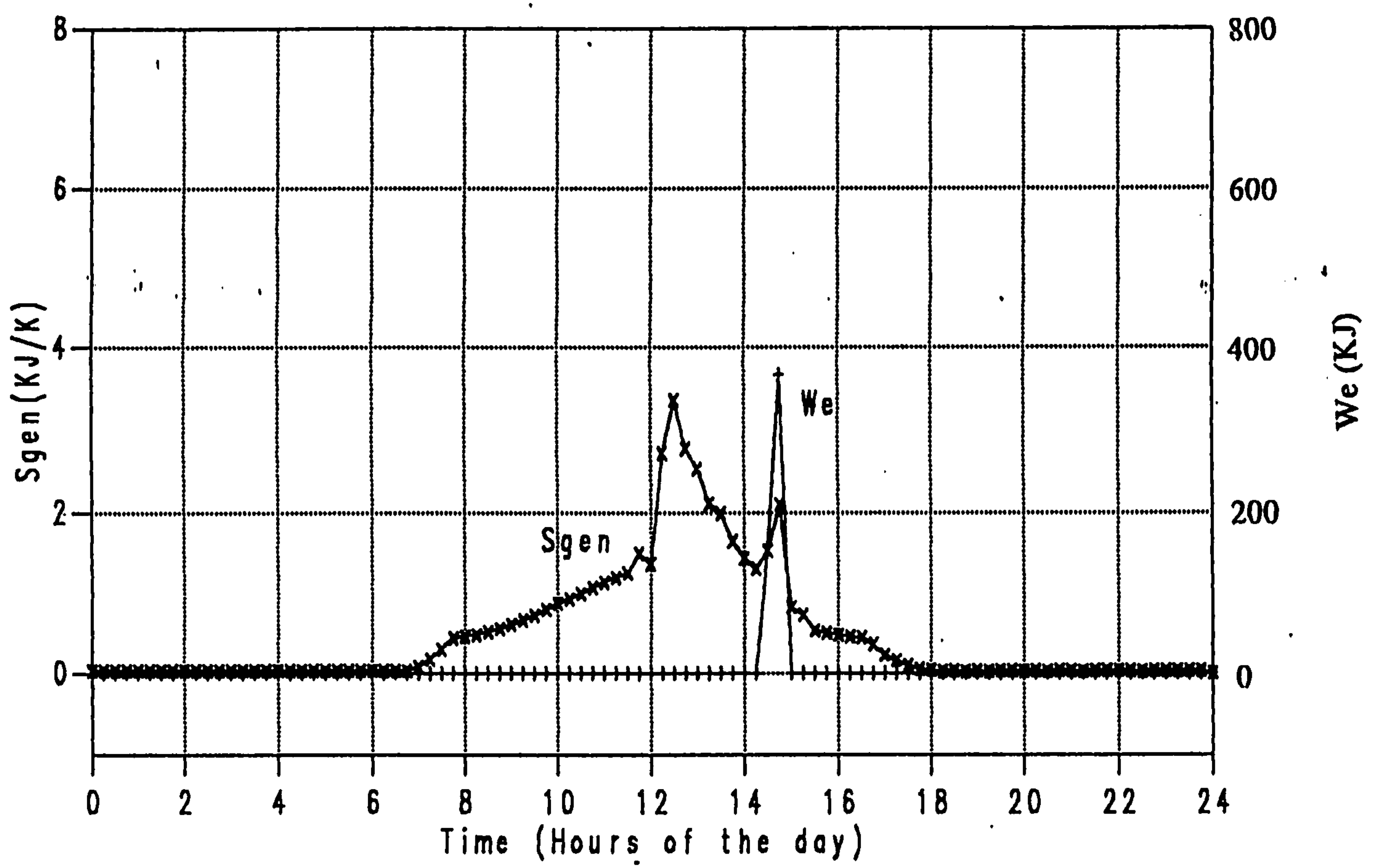



Fig. 5.10 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 50, M/Vs = 1.0, Optimum load Profile)

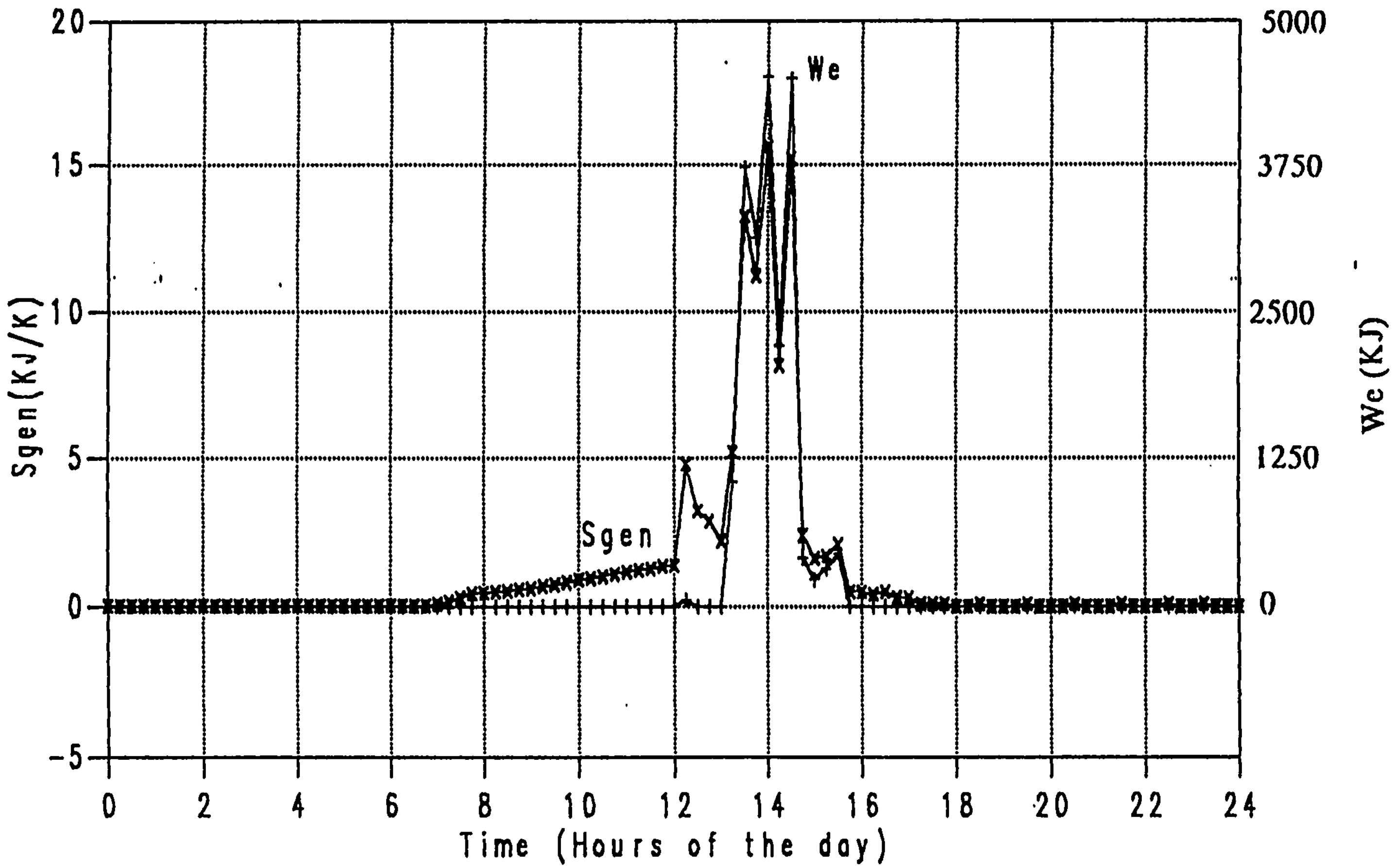



Fig. 5.11 Variation of Entropy and Auxiliary Energy
(Vs/Ac = 50, M/Vs = 1.5, Optimum load Profile)

**TABLE (5.1) Total Auxiliary Energy Consumption and Total Entropy Generation (For Optimum Load Profile) For System#1**

| Case | $V_s$ (lts) | $V_s/A_c$ | $M/V_s$ | $S_{gen}$ (KJ/°K) | Aux Heat (KJ) |
|---|---|---|---|---|---|
| 1 | 250 | 50 | 0.5 | 44.85 | 0.0 |
| 2 | " | " | 1.0 | 50.40 | 524.88 |
| 3 | " | " | 1.5 | 109.70 | 20565.0 |
| 4 | 350 | 70 | 0.5 | 46.64 | 0.0 |
| 5 | " | " | 1.0 | 100.60 | 17500.5 |
| 6 | " | " | 1.5 | 184.80 | 44451.0 |
| 7 | 450 | 90 | 0.5 | 47.82 | 0.0 |
| 8 | " | " | 1.0 | 152.20 | 34047.0 |
| 9 | " | " | 1.5 | 266.20 | 69786.0 |

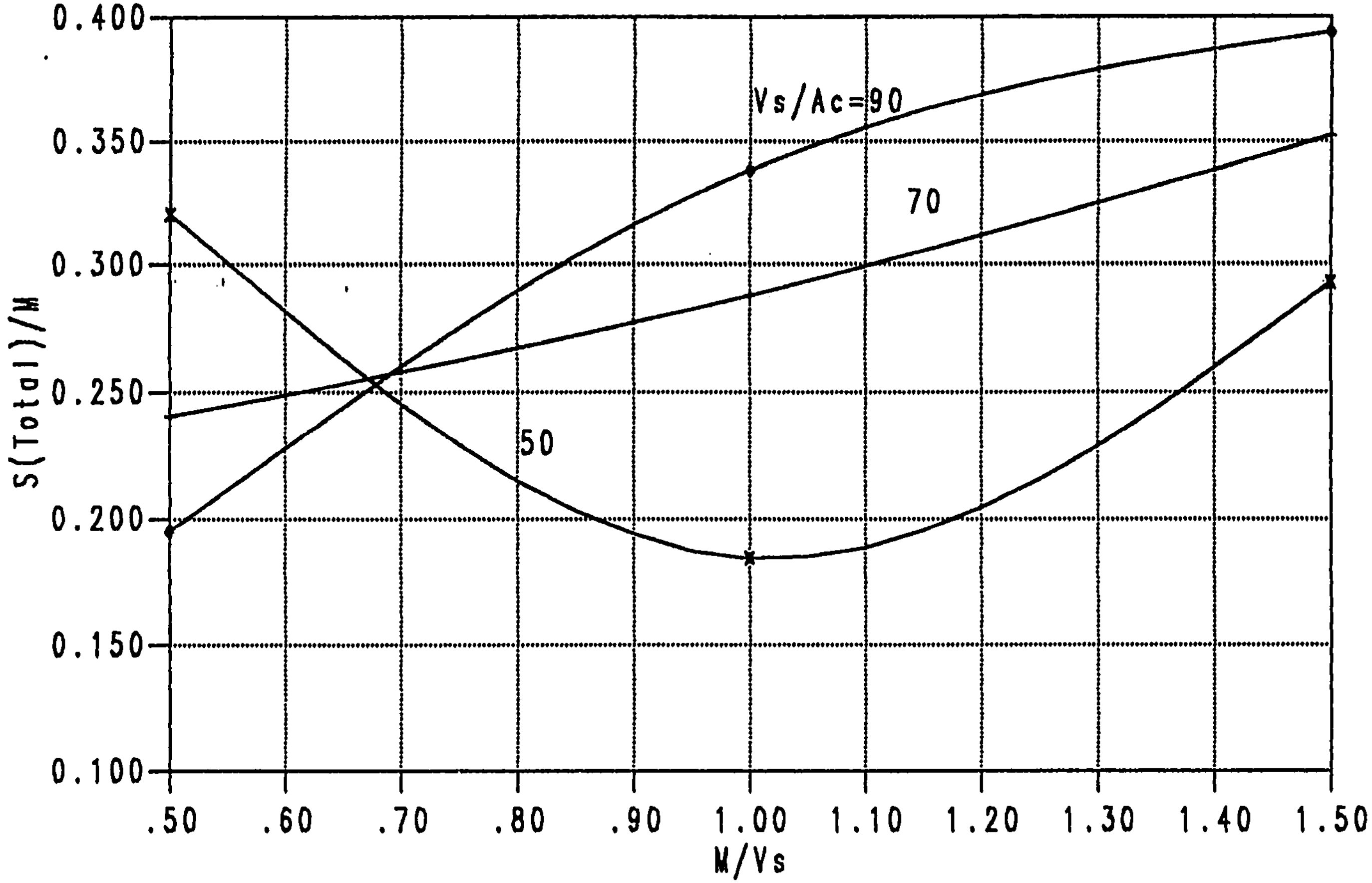



Fig. 5.12 Total Entropy generation per liter of water withdrawal (Optimum load profile)

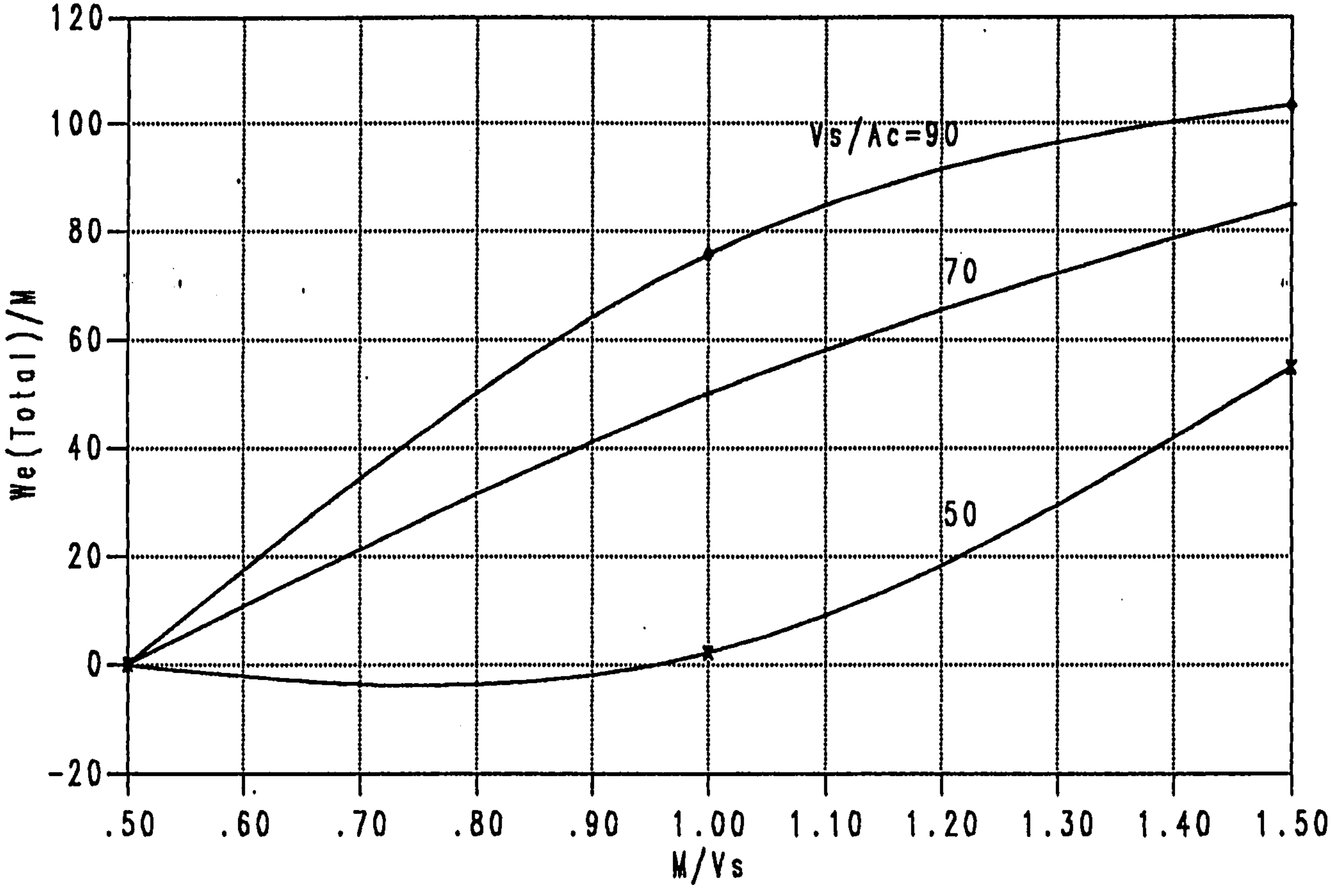



Fig. 5.13 Total Auxiliary energy per liter of water withdrawal (Optimum load profile)

**5.6 Entropy minimization for system#2 via Dynamic Programming:**

The equations for dynamic programming solution of system #2 are as given below:

$$T_{sn} = T_{so} + \frac{\Delta t}{M_s C_s}\Big\{A_c F_R[Q_{sol} - U_L(T_s - T_a)] - A_s U_s(T_s - T_a)$$

$$- m_l C_p(T_s - T_a)\Big\} \qquad (5.27)$$

$$y_n = y_o + \Delta t \times m_L$$

where

$T_{so}$ value of $T_s$ at the beginning of time step

$T_{sn}$ value of $T_s$ at the end of time step

$y_o$ Total load mass flow at the beginning of time step

$y_n$ Total load mass flow at the end of time step

with boundary conditions:

$T_o(0) = 333°K$

$T_s(T)$ = constant determined by simulation results

$y(0) = 0$

$y(T) = M =$ Total mass flow determined simulation result

4. The performance criteria Eq.(5.17) which we seek to minimize is discretized as:

$$I = \sum_{k=1}^{K}\Big\{ - \frac{Q_{sol}}{A_c} + \frac{A_c U_L (T_s - T_a)}{T_a} + m_c C_p \ln(\frac{T_c}{T_s})$$

$$- A_s U_s \frac{(T_s - T_a)}{T_s} - m_L C_p \frac{(T_s - T_L)}{T_s} + \quad + A_s U_s \frac{(T_s - T_a)}{T_a}$$

$$+ m_L C_p \ln(\frac{T_s}{T_l}) + m_L C_p \ln(\frac{T_D}{T_s}) \Big\} \times \Delta t$$

$$= \sum_{k=1}^{K} L(m_1, m_2) \Delta t \tag{5.28}$$

All the variables and parameters are as defined in the previous section. Optmization of system#2 is carried out using the procedure as explained in optimization of system#1.

Several optimization runs are made for those $V_s/A_c$ and $M/V_s$ ratios for which simulation has already been done using Rand profile. For each optimization run, the boundary conditions are taken from the corresponding Rand profile simulation results.

A typical optimum load profile for a given tank volume and heating load is shown in Fig.(5.14). Heating load is confined during afternoon hours to minimize entropy generation and auxiliary energy consumption.

A typical entropy generation variation and auxiliary energy consumption plot over a period of 24-hours for $V_s/A_c = 50$ and $M/V_s = 0.5$ is shown in Fig.(5.15). Entropy generation is high at peaks of the optimum load profile, and no auxiliary energy is needed for the case under consideration.

Temperature variations for $V_s/A_c = 50$ and $M/V_s = 0.5, 1.0, 1.5$ over a period of 24-hours are shown in Fig.(5.16). As expected temperature decreases with increase in heating loads.

A detail breakdown of the optimization results for different tank sizes and heating loads is furnished in Table (5.2). For smaller tanks, increase in loads, increases entropy generation and auxiliary energy consumption by small amounts, and the increase is more for larger tanks.

Critical examination of Fig.(5.17) of entropy generation per liter of water drawn versus $M/V_s$ ratio for $V_s/A_c = 50, 70, 90$ yields a few interesting facts.

The minimum entropy condition corresponds to heating loads $M/V_s = 1.0$, for larger tank size of $V_s/A_c = 90$, which is opposite to the system#1. For smaller ratios of $V_s/A_c$, the minimum corresponds to higher values of $M/V_s$.

Plot of auxiliary energy consumption per liter of water versus $M/V_s$ ratios for various $V_s/A_c$ ratios indicate the opposite. The minimum point accurs for $V_s/A_c = 50$ and approximately at $M/V_s = 1.0$ as shown in Fig.(5.18). For larger values of $V_s/A_c$, the minimum tends towards lower values of $M/V_s$.

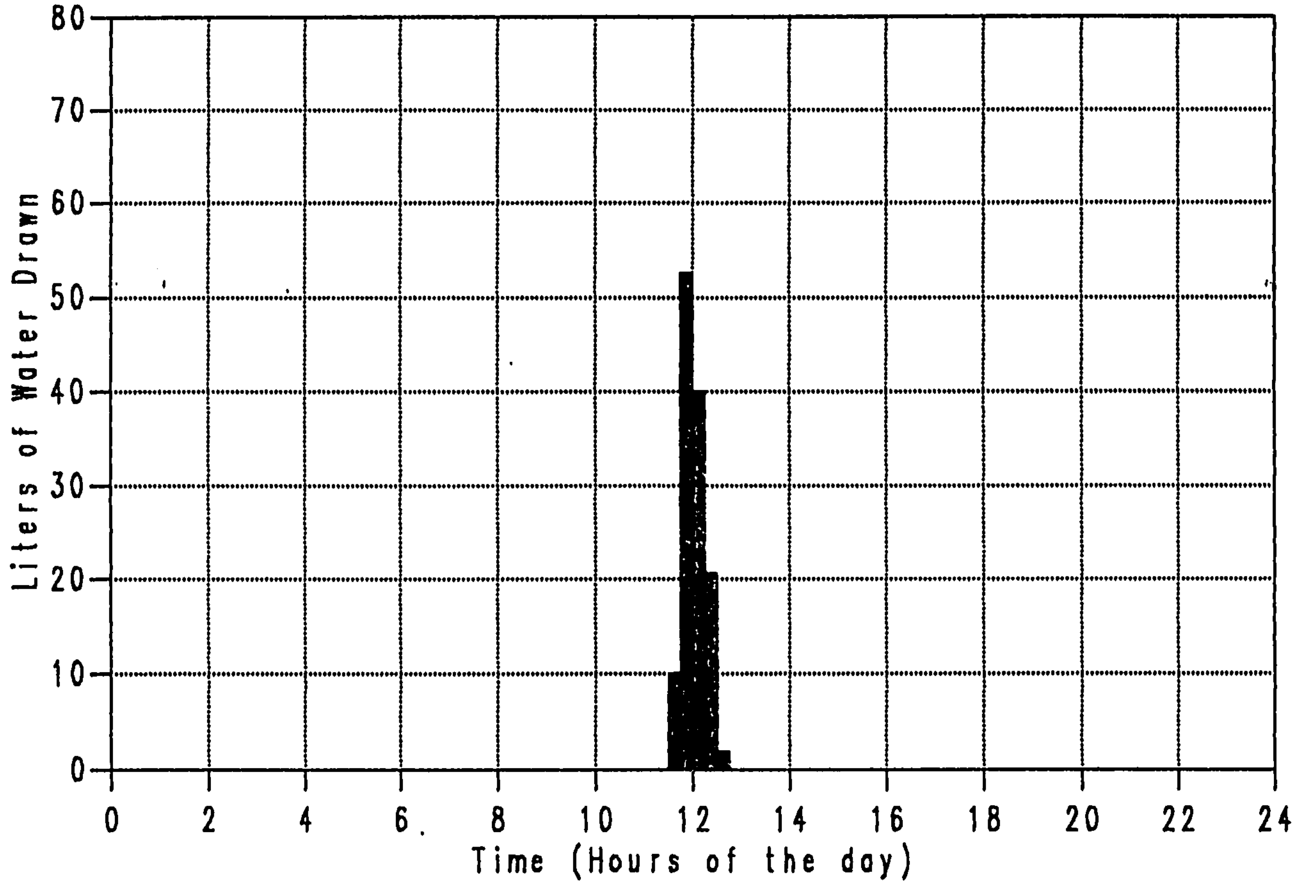



Fig. 5.14 Optimum load profile (System#2, Vs/Ac = 50, M/Vs = 0.5)

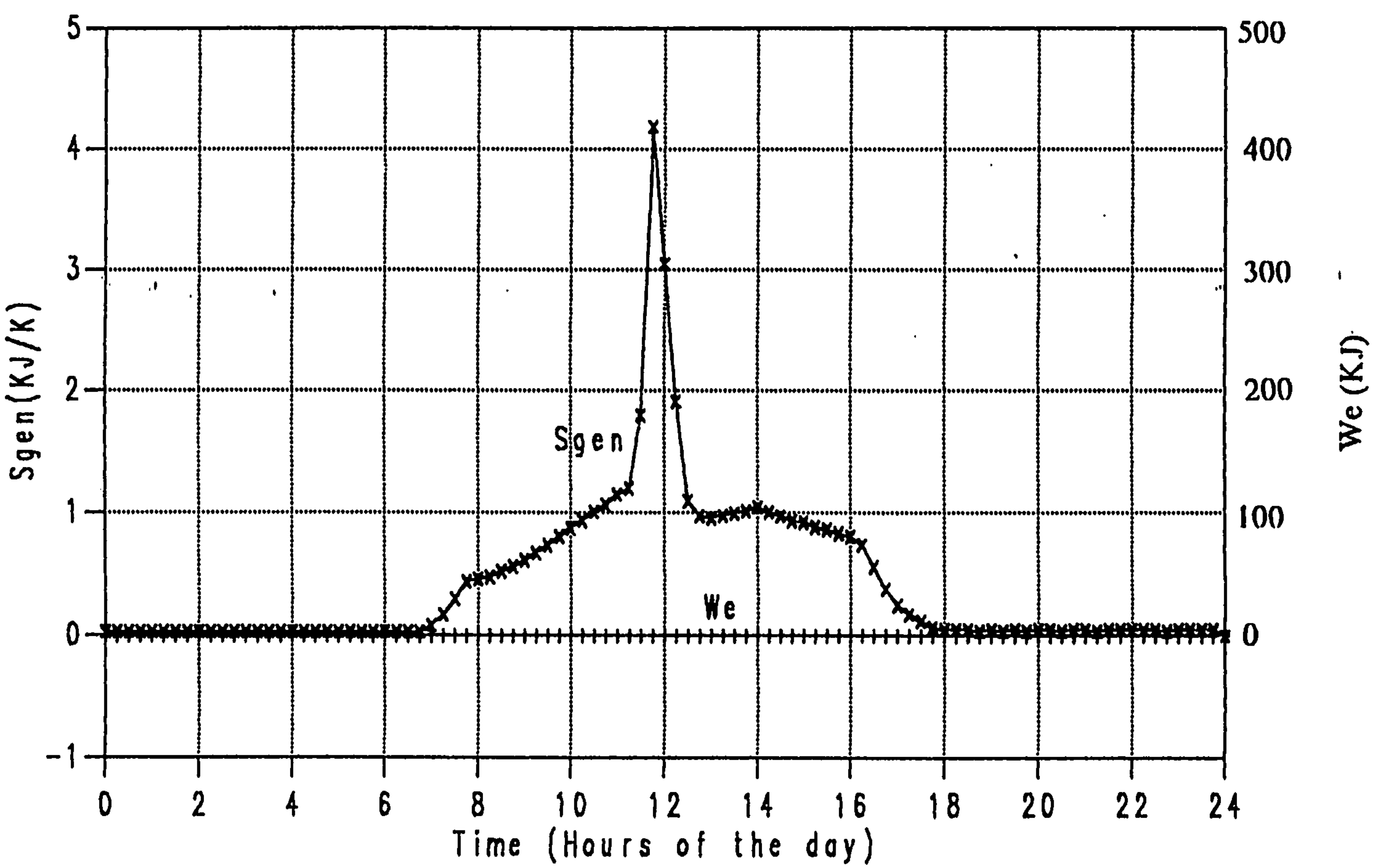



Fig. 5.15 Variation of Entropy and Auxiliary Energy
(Vs/Ac=50, M/Vs=0.5, Optimum load Profile)

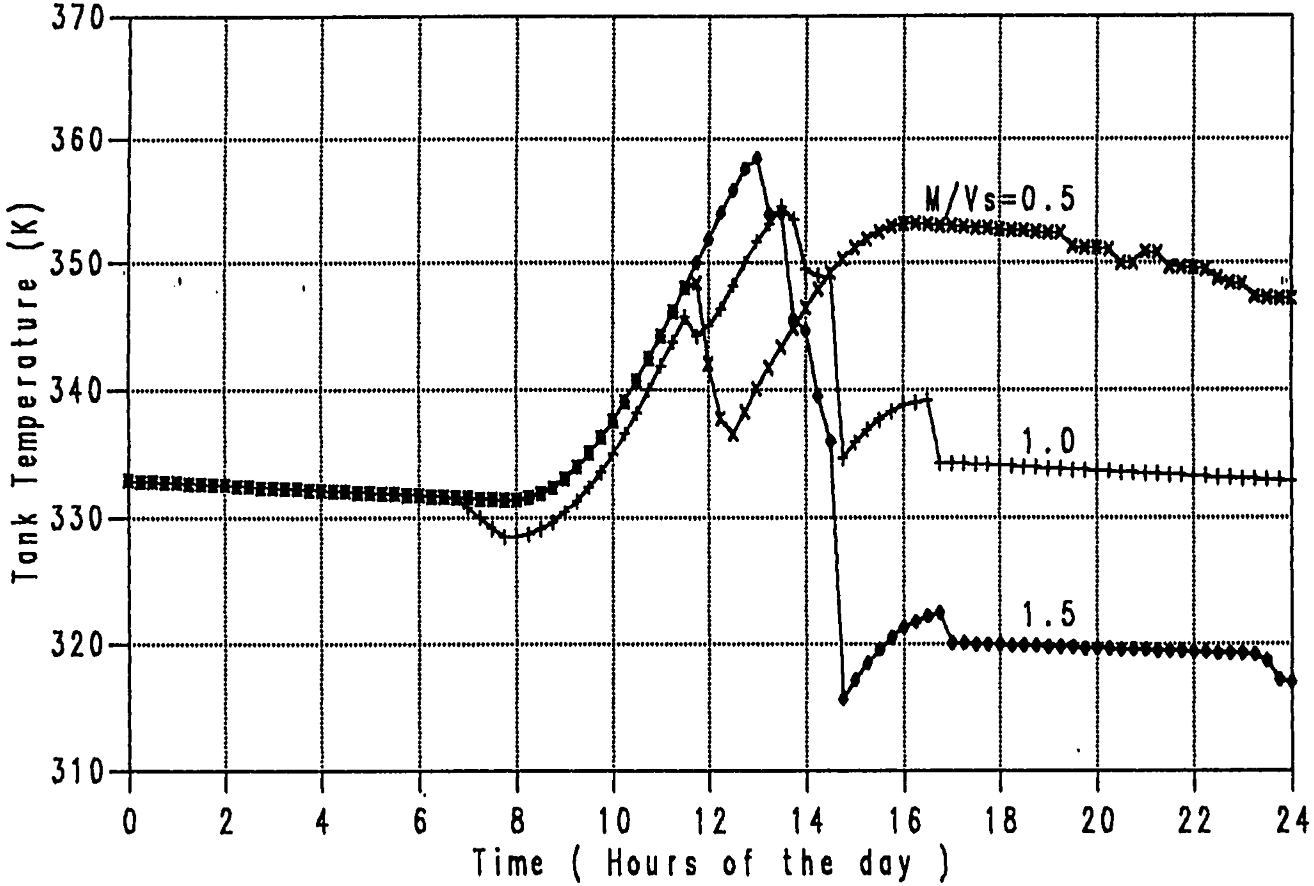



Fig. 5.16 Variation of Tank Temperature with Load
(Vs/Ac=50, Optimum load profile)

TABLE (5.2) Total Auxiliary Energy Consumption and Total Entropy Generation (For Optimum Load Profile)

For system#2

| Case | $V_s$ (lts) | $V_s/A_c$ | $M/V_s$ | $S_{gen}$ (KJ/°K) | Aux Heat (KJ) |
|---|---|---|---|---|---|
| 1 | 250 | 50 | 0.5 | 41.37 | 0.0 |
| 2 | " | " | 1.0 | 47.20 | 215.64 |
| 3 | " | " | 1.5 | 55.6 | 2190.60 |
| 4 | 350 | 70 | 0.5 | 42.71 | 102.33 |
| 5 | " | " | 1.0 | 50.71 | 1239.3 |
| 6 | " | " | 1.5 | 77.71 | 11736.0 |
| 7 | 450 | 90 | 0.5 | 43.24 | 119.8 |
| 8 | " | " | 1.0 | 59.67 | 5577.30 |
| 9 | " | " | 1.5 | 114.50 | 23400.0 |

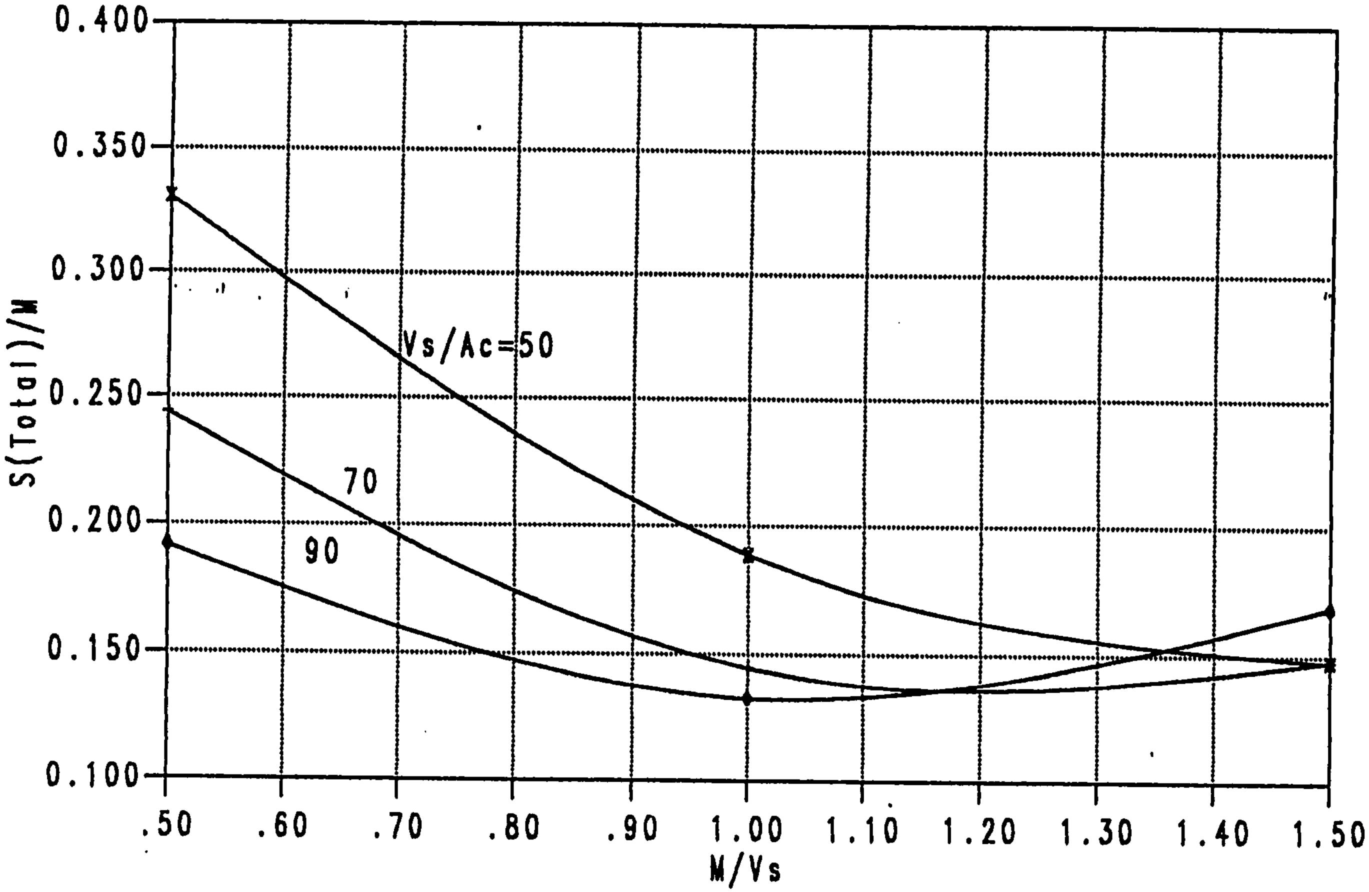



Fig. 5.17 Total Entropy generation per liter of water withdrawal (Optimum load profile)

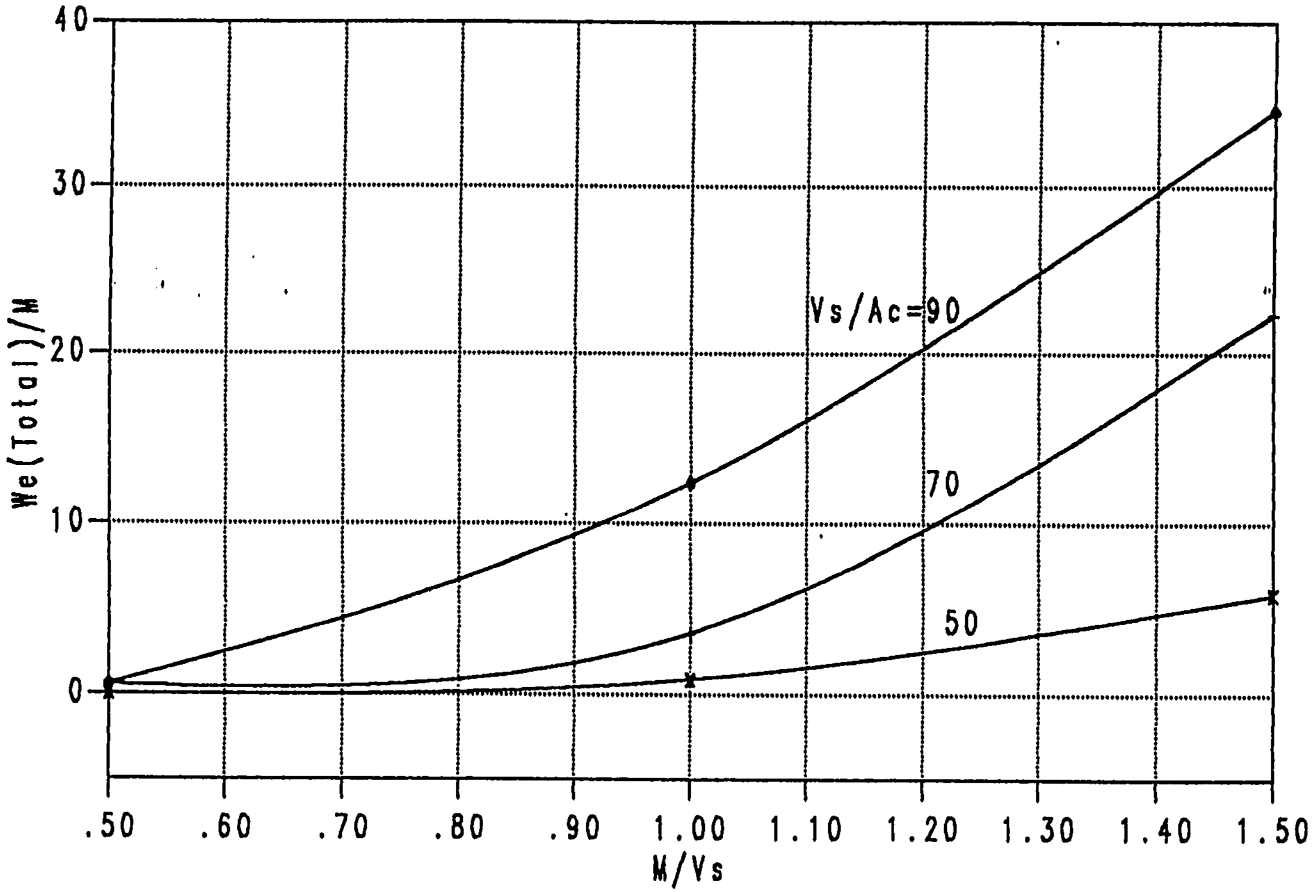



Fig. 5.18 Total Auxiliary Energy per liter of water withdrawal (Optimum load profile)

### 5.7 Comparison of Simulation and Optimization Results:

Comparison of total entropy generation and total auxiliary energy consumption of system#1, from simulation and optimization results, as listed in Tables (4.2, 4.3, 5.1, 5.2), leads to the following conclusions:

(1). Substantial savings in auxiliary energy and a corresponding decrease in entropy generation are observed in optimization results. The optimum point in simulation is obtained from Rand profile results for $V_s/A_c = 50$, and $M/V_s = 1.0$. The optimization results also indicate the optimum point for $V_s/A_c = 50$, and $M/V_s = 1.0$. For the purpose of comparison the total auxiliary energy consumption and total entropy generation for this particular case are given in Table (5.3).

As it can be seen from the Table (5.3), the optimum load profile, as compared to Rand profile, generates 28.7% less entropy, while needing 92.7% less auxiliary energy.

(2) Both simulation and optimization results [Figures (4.26-4.27, 5.12-5.13)] state that for smaller tank volumes, the minimum entropy creation and auxiliary energy consumption corresponds to a total mass flow which is approximately equal to the tank volume. However, the minimum shifts towards lower $M/V_s$ ratios for larger tank volumes.

(3) Comparison of total entropy generated and total auxiliary energy consumed of system#2, from simulation and optimization results as given in Tables (4.3, 5.2), states that the a system supplying load in accordance with a optimum load profile as compared to Rand profile results generates less entropy and consumes

TABLE (5.3) Comparison of Simulation and Optimization results

| | $V_s$ (lts) | $V_s/A_c$ | $M/V_s$ | $S_{gen}$(Total) KJ/°K | Aux (Total) Heat (KJ) |
|---|---|---|---|---|---|
| Simulation | 250. | 50. | 1.0 | 70.68 | 7183.10 |
| Optimization | " | " | " | 50.40 | 524.88 |

TABLE (5.4) Comparison of Simulation and Optimization results

| | $V_s$ (lts) | $V_s/A_c$ | $M/V_s$ | $S_{gen}$(Total) KJ/°K | Aux (Total) Heat (KJ) |
|---|---|---|---|---|---|
| Simulation | 450. | 90. | 1.0 | 66.79 | 7732.60 |
| Optimization | " | " | " | 59.67 | 5577.3 |

TABLE (5.5) Comparison of Optimization results

| | $V_s$ (lts) | $V_s/A_c$ | $M/V_s$ | $S_{gen}$(Total) KJ/°K | Aux (Total) Heat (KJ) |
|---|---|---|---|---|---|
| System#1 | 450. | 90. | 1.0 | 152.20 | 34047.0 |
| System#2 | " | " | " | 59.67 | 5577.3 |

less auxiliary energy.

Table (5.4) shows a comparison of total entropy generation and total auxiliary energy consumption for a particular case $V_s/A_c = 90$, and $M/V_s = 1.0$.

It can be seen from Table (5.4), that the optimum load profile generates 11.9% less entropy while needing 38.6% less auxiliary energy, as compared to Rand profile.

**5.8 Comparison of two Optimized Systems:**

Comparison of the two optimum systems from the point of view of entropy generation and auxiliary energy consumption is clear from Tables (5.1, 5.2). Optimum system#2 as compared to optimum#1 gives better performance from the stand point of second and first law.

Table (5.5) shows comparison of two optimum systems for $V_s/A_c = 90$ and $M/V_s = 1.0$.

Table(5.5) demonstrates that optimum system#2 generates 61% less entropy and consumes 83.6% less auxiliary energy.

Results of optimum system#1 Figures(5.12, 5.13) indicate that a tank volume to collector area ratio of 50, for a heating load approximately equal to tank size gives total minimum entropy generation and total minimum auxiliary energy consumption per kg of water withdrawn.

Results of optimum system#2 Figures(5.17) indicate that a tank volume to collector area ratio of 50 gives total minimum entropy per liter of water for higher heating loads. However Fig.(5.18) points that for minimum auxiliary energy, for $V_s/A_c$ ratio of 50, $M/V_s$ ratio should be equal to one, approximately.

# CHAPTER 6

# CONCLUSIONS AND RECOMMENDATIONS

## 6.1 Conclusions

To understand how solar energy can be used most effectively to satisfy specific needs, two typical Solar Domestic Hot Water Systems were analyzed on the basis of second law of thermodynamics and the systems were optimized for minimum entropy generation and minimum auxiliary heat consumption. The systems were subjected to dynamic, thermal and total load mass flow constraints. Parametric studies were performed to examine critically, the sensitivity of system performance to variations in the system parameters. The following conclusions are deduced from this investigation.

1. Simulation results of system#1 subject to three hourly load profiles Rand, ASHRAE and Vitro, demonstrate that a system subjected to Rand profile, in general provides better results from both first and second law point of view.

2. Simulation results of sytem#2 subject to three hourly load profiles Rand, ASHRAE and Vitro, state that results of Rand and ASHRAE profiles exhibit lot of similarity from the point of view of second law and first law. Also, the entropy generated and auxiliary energy consumed are considerably less for Rand and ASHRAE profiles as compared to Vitro profile.

3. Storage tank size and heating load for a constant collector area have a significant effect on the system performance, increase in either of the

two, appreciably increases the auxiliary energy consumption and entropy generation.

4. In general both simulation and optimization results indicate that for minimum entropy and minimum auxiliary energy consumption the hot water withdrawal should be approximately equal to tank size for one day's data.

5. Optimization of SDHW systems for minimum entropy generation, show considerable reduction in entropy generation and auxiliary energy requirement as compared to simulation results.

   For a ratio of $V_s/A_c = 50$ and $M/V_s = 1.0$ , system#1 using optimum load profile generates 28.7% less entropy while consuming 92.7% less auxiliary energy as compared to same system subjected to Rand load profile.

6. Simulation and optimization results indicate that provision of auxiliary heater in series with storage system as compared to backup heater in the storage tank, exhibits better first and second law performance. Comparison of optimization results of the two systems for $V_s/A_c = 90$, and, $M/V_s = 1.0$ shows that system#2 generates 61% less entropy and consumes 83.6% less auxiliary energy as compared to system#1.

7. Although the optimum load profiles may not be applied practically, but they show that there is room for reductions in entropy generation and auxiliary consumption, if there could be a way of utilizing these profiles. However both simulation and optimization studies suggest indicate a

practical way of deciding tank size, and collector area for a required hot water load. For example, A load of 500 liters requires a tank size of 500 liters for optimum operation.

**Recommendations for Future Work**

1. Collectors are often used in combination with a heat exchanger between collector and storage allowing the use of antifreeze solutions in the collector loop. So it is recommended to incorporate a heat exchanger in the systems used in this investigation and optimize the complete systems for minimum entropy generation and minimum auxiliary heat consumption subject to dynamic, thermal, and total heating load requirement constraints.

2. Second law analysis and optimal control strategy can be employed to Natural Convection (thermosyphon) water heating systems.

3. Practically, water tanks may operate with significant degrees of stratification. So it is recommended to consider the stratification effects in tanks and analyze and optimize SDHW systems on the basis of second law of thermodynamics.

4. Simulation can be done for extended periods to reinforce the findings of this investigation. For large periods, the optimal control problem could be very involved.

5. Other SDHW systems existing in the literature can be analyzed and optimized from the second law point of view.

## *REFERENCES*:


1. Kubat, L. and Zemañ, J., "Entropy and Information in Science and Philosophy", Elsevier, N.Y. 1975,
2. Shannon, C.E., "A Mathematical Theory of Communication", The Bell System Technical Journal, Vol 27, pp. 379-623, 1948.
3. Nicholas Georgescu-Roegen, "The Entropy Law and the Economic Process", Harward University Press, 1971.
4. Nicholis, G. and Prigogine, I. "Self-Organization in Non-Equilibrium Systems", John wiley, 1977.
5. Scilliano, A.V., "Entropy Considerations in Environmental Planning", University of California, Berkeley, Monograph No. 25, Inst. of Urban and Regional Development, 1976.
6. Arnheim, R., "Entropy and Art", University of California, Berkeley Press, 1971.
7. Odam, H., "Environment, Power and Society", John Wiley, New York, 1971.
8. Murphy, G.W., et-al., "The Minimum Energy Requirements for Sea Water Conversion Processes", US Dept. of Interior, Saline Water Research and Development Progress Report No: 5, 1956
9. Mitsch, W.J., et-al, "Energetics and Systems", Ann Arbor Science Publications, Michigan, U.S.A, 1982.
10. Keenan, J.H., "Thermodynamics", MIT Press, 1941.
11. Bejan, A., "Entropy Generation Through Heat and Fluid Flow", John Wiley, New York, pp. 32, 1982.
12. Moran, M.J., "Availability Analysis", Prentice Hall, New Jersey, 1982.

13. Ahern,J.E., The Exergy Method of Energy System Analysis, Wiley, New York (1980).

14. Moore, D.F., Thermodynamic Principles of Energy Degrading, McMillan, 1980.

15. Sussman, M.V, "Availibility (Exergy) Analysis: A self-instruction Manual," M.V. Sussman, Publisher, Department of Chemical Engineering, Tufts University, Medford, MA (1980).

16. Rathore, R.N.S. and Kenney, W.F., Thermodynamic Analysis for Improve Energy Efficiency, AICHE Today Series, AICHE, New York, (1980).

17. Bejan, A., Second Law Analysis in Heat Transfer and Thermal Design, Chapter 1 in Advances in Heat Transfer, Vol. 15, pp. 1-58 (1982).

18. Bejan, A., Entropy Generation Through Heat and Fluid Flow, Wiley, New York (1982).

19. Moran, M.J., Availability Analysis - A Guide to Efficient Energy Use, Prentice-Hall, Englewood Cliffs, NJ (1982).

20. Seader, J.D., Thermodynamic Efficiency of Chemical Processes, MIT Press, Cambridge, MA (1982).

21. Liu, Y. A. and Wepfer. W. J., Theory and applications of second law analysis: a bibliography, chapter 18 in Thermodynamics: Second Law Analysis, Vol. II, R. Gagioli, ed., American Chemical Society Symposium Series, ACS, Washington, D.C. (1983).

22. Collares, et. al.,"Design and Optimization of Solar Industrial Hot Water Systems with Storage", Solar Energy, Vol 32, (1984), p. 121-133.

23. Gordon, J.M. and Rabl, A., "Design Analysis and Optimization of

Solar Industrial Process Heat Plants Without Storage", Solar Energy, Vol 28, (1982), No 6, p. 519-530.

24. Harold, et. al, "Control Strategies for Solar Heating/Cooling Systems That Reduce Electric Utility Peak Loads, Proc. 1978, Ann, Meas, AS/ISES.
25. Wijeysundera, N.E. and Ho, J.C., Long term Thermodynamic Performance of Solar Heat and Power Systems,Applied Energy, Vol 24, (1986), p. 43-63.
26. Duffie, J.A. and Beckmann, W.A., " Solar Engineering of Thermal Processes, John Wiley, New York, 1980, 223-225.
27. Lof, G.O.G. and Karaki, S., "System Performance For The Supply of Solar Heat", Mechanical Engineering, Vol 32, (1983).
28. Ronald, A.F. and Hunter Fanney, A., " Thermal Performance Comparisons for a Solar Hot Water System", ASHRAE Journal, Aug (1983).
29. Zollner, A., et-al., "A Performance Prediction Methodology for Integral Collection-Storage Solar Domestic Hot Water Systems", Journal of Solar Energy Engineering, Vol. 107, pp. 265-272, 1985.
30. Newell, T.A., et-al., "Successive Lumped Analysis Simulation of Solar Energy Collection Systems", Energy Research, Vol. 10, pp. 277-290, 1986.
31. Naumann, P. and Wolfson, R., "Proportional Versus On/off Control", Solar Energy Engineering, Vol. 106, Nov (1984), p. 423-427.
32. Winn, C.B., Controls in Solar Energy Systems, Advances in Solar Energy, (1983), p. 209-240.
33. MacDonald, T.E., Farris, D.R. and Melsa, J.L., "Energy Conservation

Through Adaptive Optimal Control For A Solar Heated And Cooled Building", Workshop on Control of Solar Energy Systems for Heating and Cooling, 1978.

34. Fritchman, B.M. and Grantham, W.J., "Optimal Control of a Bilinear Solar Collector/Heat Exchanger System", Proc. American Control Conference, 1155-1157, 1982.

35. Auslander, D.M., et-al., "An Optimal Standard for Solar Heating Systems", Transactions of the ASME, Vol 101, pp. 138-149, 1979.

36. Winn, C.B. and Winn, R.C., "Optimal Control of Auxiliary Heating and Off-Peak Energy Storage Installations", Solar Energy Engineering, Vol. 105 Feb (1983), No. 1, p. 50-57.

37. Winn, R.C. and Winn, C.B., "Optimal Control of Auxiliary Heating of Passive Solar Heated Buildings", Solar Energy, Vol. 35, (1985), p. 419-427.

38. Dorato, P., "A Review of the Applications of Modern Control Theory to Solar Energy Systems", Proc. IEEE 7th Decision and Control Conferences, (1979), p. 907-910.

39. Kovarik, M. and Lesse, P.F., "Optimal Control of Flow in Low Temperature Solar Collectors", Solar Energy, Vol. 18, (1976), p. 431-435.

40. Winn, R.C. and Winn, C.B., "Optimal Control of Mass Flow Rate Solar Collectors", Solar Energy Engineering, Vol. 103, May (1981), p. 113-120.

41. Saltiel, C. and Sokov, M., " Optimal Control of a Multicomponent Solar Collector Systems", Solar Energy, Vol. 34, No. 6, 463-473,1985.

42. Orbach, A., Rischl, R., Herczfeld, P.R. and Konyk Jr, S., "Optimal and Sub-optimal Control Strategies and Sensitivity Study For Solar

Liquid Collector Systems", Proc. ISES, Atlanta, Vol. 2, 1421-1425, 1985.

43. Chato, D.J. and Newell, T.A:, "An Investigation of Optimal Process Load Schedules for Solar Batch Water Heaters", Proceedings ASME Solar Energy Conference, Anaheim, 1986.

44. Gribik, J. A. and Osterle, J.F., The Second Law Efficiency of Solar Energy Conversion, Solar Energy Engineering, 106 (Feb. 1984), p. 16-20.

45. Bejan, A., Kearney, D.W. and Kreith, F., Second Law Analysis and synthesis of solar collectors, Solar Energy Engineering, 103 (Feb. 1981),p.23.

46. Bejan, A., Extraction of Exergy from Solar Collectors under Time Varying Conditions, Int. Journal of Heat & Fluid Flow, Vol 3, June (1982), p. 67.

47. Fujiwara, M., Exergy Analysis for the Performance of Solar Collectors, Solar Energy Engineering, 105 (May 1983), p.163.

48. Chelghoum, D.E. and Bejan, A., Second Law Analysis of Solar Collectors With Energy Storage Capability, Solar Energy Engineering, Vol. 107, Aug (1985), p. 244-251.

49. Kar, A., Exergy Efficiency and Optimum Operation of Solar Collectors, Applied Energy, 21 (1985), 301-314.

50. Kar, A., Exergy Optimization of Flow Rates in Flat Plate Collectors, Int. Journal of Energy & Research, Vol 13, No 3, 1989.

51. Kar, A. and Shaahid, S.M., Optimum Exergy Efficiency of Solar Collectors, Int. Journal of Energy Research, Vol. 13, No 4, 1989.

52. Rosen, M.A., et-al., "Exergy Analysis for the Evaluation of the

Performance of Closed Thermal Energy Storage Systems", Journal of Solar Energy Engineering, Vol. 110, pp. 255-261, 1988.

53. Kalaba, R. and Spingarn, K., "Control Identification and Input Optimization", Plenum Press, New York, 1982.

54. Sage, A.P. and White, III, CC., "Optimum Systems Control", Prentice-Hall, N.J., 1977.

55. Larson, Robert, E. and Casti. John, E., "Principles of Dynamic Programming", Marcel Dekker, New York, 1978.

56. Gutierrez, G., Hincapie, F., Duffie, J.A., and Beckman, W.A., "Simulation of Forced Circulation Water Heaters; Effects of Auxiliary Energy Supply, Load Type and Storage Capacity", Solar Energy 15, 287, 1974.

# APPENDICES

# APPENDIX A

## Simulation program for System#1

```
C----------------------------------------------------------------
C   21 -2 1989
C----------------------------------------------------------------
C THIS PROGRAM IS TAILORED TO REPRESENT SYSTEM#1
C IFLAG IS USED TO IDENTIFY ITERATION NO
C NOTE: IN THIS PROGRAM UNIT OF TIME IS SECS
C YI ---- STORAGE TANK TEMPERATURE (DEGREE KELVIN)
C YP ---- PLATE TEMPERATURE (DEGREE KELVIN)
C TC ---- COLLECTOR TEMPERATURE (DEGREE KELVIN)
C DELT --- TIME STEP IN SECS
C AC  --- COLLECTOR AREA
C MC  --- COLLECTOR FLOW RATE
C V   --- STORAGE TANK VOLUME
C FR = 0.88 (APPROXIMATELY)
C As = SURFACE AREA OF THE STORAGE TANK
C Qs = SOLAR RADIATION
C Ta = AMBIENT TEMPERATURE (K)
C Ms = MASS OF STORAGE TANK (KG)
C UO(J)--- LOAD FLOW RATE (LITERS/S)
C SUMMATION OF UO = M (TOTAL LOAD FLOW)
C WE  --- AUXILIARY HEAT TO BE ADDED TO MEET THE LOAD
C SC  --- ENTROPY GENERATION IN THE COLLECTOR
C SS  --- ENTROPY GENERATION IN STORAGE TANK
C SM. --- ENTROPY GENERATION DUE TO MIXING
C----------------------------------------------------------------
      IMPLICIT REAL *8 (A-H,O-Z)
      REAL *8 MS, MC
      DIMENSION TA(100),QS(100),QS1(100),TC(100),UO(100),YI(100),
     & SC(100),SS(100),TT(100),WE(100),UT(100),SM(100),
     & YP(100),UTN(100),STOT(100),STT(100)
C--------------------------------------------------------
      AC=5.00
      CP=4190.
      CS=4190.
      UL=4.
      US=0.5
C-----------------
      V = 250.
C     V = 350.
C     V = 450.
C---------
      AS=6.0*(V/1000.)**(2./3.)
C-----------------
      MS=V
      TL = 273.+20.
      TM = 273.+20.
      TD = 273.+60.
      YI(1) = 333.
      DELT = 900.
      MC=0.10
      FR1=MC*CP/(AC*UL)
```

```
      FR = FRI*(1.-DEXP(-0.9/FRI))
C-------------------------------------------------------
      WRITE(6,988)
988  FORMAT(4X,'VITRO PROFILE'//)
      WRITE(6,989)
989  FORMAT(4X,'VS=450.      VS/AC = 90.      M/VS =1.5')
C-------------------------------------------------------
C     INITIALIZATION OF UXT IS TO FIND THE TOTAL LOAD MASS FLOW
C-------------------------------------------------------
C     UXT =0.0
C-----------------------
      DO 11 J = 1,97
      READ(5,*)TT(J),QS(J),TA(J),UO(J)
C-------------------------------------------------------
C  TO FIND THE TOTAL LOAD FLOW  (UXT)
C-------------------------------------------------------
C    UO(J) = (UO(J)/3600.)
C    UXT = UXT + UO(J)*900.
C----------
      UO(J) = 0.5*(UO(J)/3600.)*(250./269.)
C    UO(J) = 1.0*(UO(J)/3600.)*(250./269.)
C    UO(J) = 1.5*(UO(J)/3600.)*(250./269.)
C-------------
C    UO(J) = 0.5*(UO(J)/3600.)*(350./269.)
C    UO(J) = 1.0*(UO(J)/3600.)*(350./269.)
C    UO(J) = 1.5*(UO(J)/3600.)*(350./269.)
C---------------
C    UO(J) = 0.5*(UO(J)/3600.)*(450./269.)
C    UO(J) = 1.0*(UO(J)/3600.)*(450./269.)
C    UO(J) = 1.5*(UO(J)/3600.)*(450./269.)
C-------------------------------------
 11  CONTINUE
C----------------------------
C    WRITE(6,*)UXT
C----------------------------
      QS1(1) = QS(1) -UL*(Y1(1)-TA(1))
      IF(QS1(1) .LE. 0.0)QS1(1)=0.0
      WE(1) = 0.0
      WETOT = 0.0
      WRITE(6,99)
 99  FORMAT(//2X,'TIME',10X, 'Y1',13X,'QS1',14X,'WE',14X,'WETOT'//)
C-------------------------------------
      DO 10 J = 2,97
      UT(J) = UO(J)
      WE(J) = AS*US*(Y1(J-1)-TA(J-1)) + UT(J)*CP*(Y1(J-1)-TL)
     &     -AC*FR*QS1(J-1)
      IF(WE(J) .LT. 0.0) WE(J)=0.0
      IF(Y1(J-1) .GT. 333.) WE(J)=0.0
      IFLAG = 1
  6  Y1(J) = Y1(J-1)+ (DELT/(MS*CS))*( AC*FR*QS1(J-1)
     &     - AS*US*(Y1(J-1)-TA(J-1)) - UT(J)*CP*(Y1(J-1)-TL)
     &     + WE(J) )
      IF((Y1(J)-333.) .LE. 0.01) GOTO 5
      GOTO 15
C-------------------------------------------------------
C  IF (Y1-333.) IS LESS THAN OR EQUAL .01 MORE WE  IS ADDED TO
```

```
C INCREASE Y1 TO 333
C-----------------------------------------------------------
 5 QS1(J) = QS(J)-UL*(Y1(J)-TA(J))
      IF(QS1(J) .LE. 0.0)QS1(J)=0.0
      WE(J) = (333.-Y1(J))*MS*CS/DELT -AC*FR*QS1(J-1)
     &     + AS*US*(Y1(J-1)-TA(J-1)) + UT(J)*CP*(Y1(J-1)-TL)
      Y1(J) = 333.
      IF(WE(J) .LE. 0.0) WE(J)=0.0
      WETOT = WETOT + WE(J)
C--
      WRITE(6,33)(J-1)*0.25,Y1(J),QS1(J),WE(J),WETOT
 33 FORMAT(2X,F5.2,4E16.5)
C--
C   WRITE(6,33)(J-1)*0.25,Y1(J)
C 33 FORMAT(2X,F5.2,E12.5)
      SM(J) = 0.0
      GOTO 10
C-----------------------------------------------------------
C IF (Y1-333.) IS GREATER THAN .01
C MIXING IS DONE TO REDUCE Y1 TO 333.
C-----------------------------------------------------------
15 UT(J) = UO(J)*(333.-293.)/(Y1(J)-293.)
      IF(IFLAG .EQ. 1)THEN
      UTN(J) = UT(J)
      IFLAG  = 2
      GOTO 6
      ENDIF
      IF(ABS(UTN(J)-UT(J)).GT. 1.0) THEN
      UTN(J) = UT(J)
      GOTO 6
      ENDIF
C-----------------------------------------------------------
C COMPUTATION OF MIXING ENTROPY
C-----------------------------------------------------------
      SM(J) = UT(J)*CP*DLOG(333./Y1(J))
     &      + (UO(J)-UT(J))*CP*DLOG(333./TL)
C-----------------------------------
      QS1(J) = QS(J)-UL*(Y1(J)-TA(J))
      IF(QS1(J) .LE. 0.0) QS1(J)=0.0
      WE(J) = (333.-Y1(J))*MS*CS/DELT -AC*FR*QS1(J-1)
     &   + AS*US*(Y1(J-1)-TA(J-1)) + UT(J)*CP*(Y1(J-1)-TL)
      IF(WE(J) .LT. 0.0) WE(J)=0.0
      IF(Y1(J) .GT. 333.) WE(J)=0.0
      WETOT = WETOT + WE(J)
C-----------
      WRITE(6,77)(J-1)*0.25,Y1(J),QS1(J),WE(J),WETOT
  77 FORMAT(2X,F5.2,4E12.5)
C-----------
C   WRITE(6,77)(J-1)*0.25,Y1(J)
C 77 FORMAT(2X,F5.2,E12.5)
10 CONTINUE
C---------------------------------------------
C        CALCULATION OF ENTROPY
C---------------------------------------------
      TOT1 = 0.0
      TOT3 = 0.0
```

```
      TOT2 = 0.0
      STT(1)=0.0
C -----------
      DO 20 J = 2, 97
      YP(J) = Y1(J) + QS1(J)*(1.-FR)/UL
      IF(QS1(J) .LE. 0.0)THEN
      QS1(J) = 0.0
      YP(J) = TA(J) +QS(J)/UL
      ENDIF
      SC1 = DELT*(-QS(J)*AC/YP(J))
      SC2 = DELT*(+AC*UL*(YP(J)-TA(J))/TA(J))
      SC(J) = SC1 +SC2
C -------
      SS1 = DELT*(+ AC*FR*QS1(J)/Y1(J))
      SS2 = DELT*(- US*AS*(Y1(J)-TA(J))/Y1(J))
      SS3 = DELT*(+UT(J)*CP*(TL-Y1(J))/Y1(J))
      SS4 = DELT*(+ WE(J)/Y1(J))
      SS5 = DELT*(+ AS*US*(Y1(J)-TA(J))/TA(J))
      SS6 = DELT*(+UT(J)*CP*DLOG(Y1(J)/TL) )
      SS(J)= SS1+SS2+SS3+SS4+SS5+SS6
      STOT(J) =SC(J) +SS(J) +DELT*SM(J)
      STT(J) = STT(J-1)+STOT(J)
      TOT1 = TOT1+SC(J)
      TOT2 = TOT2+SS(J)
      TOT3 = DELT*SM(J) +SC(J)+ SS(J) +TOT3
C-------------------------------------------------
C   PRINTING OF RESULTS
C-------------------------------------------------
C   WRITE(6,9)Y1(J),SC1,SC2,SS1,SS2,SS3,SS4,SS5,SS6
C 9  FORMAT(9E10.3)
C-------------------------------------------------
 20  CONTINUE
      WRITE(6,55)TOT1,TOT2,TOT3
 55  FORMAT(/4X,3E15.5)
      WRITE(6,7)
  7  FORMAT(//6X,'TIME    Y1     SC    SS    SM     STOT '//)
      DO 30 J= 2,97
      WRITE(6,44) (J-1)*0.25, Y1(J),SC(J),SS(J),SM(J),STOT(J),STT(J)
 44  FORMAT(2X,F5.2,6E12.4)
 30  CONTINUE
      STOP
      END

C-------------------------------------------------
C DATA USED IN RUNNING THE ABOVE PROGRAM IS GIVEN BELOW
C FIRST COLUMN REPRESENTS TIME IN SECONDS
C SECOND COLUMN REPRESENTS SOLAR RADIATION (WATT/Sq.m) AT A PARTICULAR TIME
C THIRD COLUMN REPRESENTS TTEMPERATURE (DEGREE KELVIN)
C FOURTH COLUMN REPRESENTS LOAD FLOW RATE
C     (FOR RAND PROFILE) IN THAT TIME STEP
C-------------------------------------------------

   0.0    0.0    282.40    6.0
 900.     0.0    282.47    6.0
1800.     0.0    282.55    6.0
2700.     0.0    282.62    6.0
```

| | | | |
|---|---|---|---|
| 3600. | 0.0 | 282.70 | 0.0 |
| 4500. | 0.0 | 282.72 | 0.0 |
| 5400. | 0.0 | 282.75 | 0.0 |
| 6300. | 0.0 | 282.77 | 0.0 |
| 7200. | 0.0 | 282.80 | 0.0 |
| 8100. | 0.0 | 282.85 | 0.0 |
| 9000. | 0.0 | 282.90 | 0.0 |
| 9900. | 0.0 | 282.95 | 0.0 |
| 10800. | 0.0 | 283.00 | 0.0 |
| 11700. | 0.0 | 283.05 | 0.0 |
| 12600. | 0.0 | 283.10 | 0.0 |
| 13500. | 0.0 | 283.15 | 0.0 |
| 14400. | 0.0 | 283.20 | 0.0 |
| 15300. | 0.0 | 283.30 | 0.0 |
| 16200. | 0.0 | 283.40 | 0.0 |
| 17100. | 0.0 | 283.50 | 0.0 |
| 18000. | 0.0 | 283.60 | 0.0 |
| 18900. | 0.0 | 283.65 | 0.0 |
| 19800. | 0.0 | 283.70 | 0.0 |
| 20700. | 0.0 | 283.75 | 0.0 |
| 21600. | 0.0 | 283.80 | 4.5 |
| 22500. | 0.0 | 283.87 | 4.5 |
| 23400. | 0.0 | 283.95 | 4.5 |
| 24300. | 0.0 | 284.02 | 4.5 |
| 25200. | 65.68 | 284.10 | 13.0 |
| 26100. | 108.26 | 284.43 | 13.0 |
| 27000. | 150.84 | 284.75 | 13.0 |
| 27900. | 193.42 | 285.07 | 13.0 |
| 28800. | 236.00 | 285.40 | 18.2 |
| 29700. | 284.40 | 285.85 | 18.2 |
| 30600. | 332.80 | 286.30 | 18.2 |
| 31500. | 381.20 | 286.75 | 18.2 |
| 32400. | 429.60 | 287.20 | 24.0 |
| 33300. | 474.60 | 287.40 | 24.0 |
| 34200. | 519.60 | 287.60 | 24.0 |
| 35100. | 564.60 | 287.80 | 24.0 |
| 36000. | 609.60 | 288.00 | 18.2 |
| 36900. | 640.00 | 288.12 | 18.2 |
| 37800. | 670.40 | 288.25 | 18.2 |
| 38700. | 700.80 | 288.37 | 18.2 |
| 39600. | 731.20 | 288.50 | 12.0 |
| 40500. | 749.20 | 288.60 | 12.0 |
| 41400. | 767.20 | 288.70 | 12.0 |
| 42300. | 785.20 | 288.80 | 12.0 |
| 43200. | 803.20 | 288.90 | 10.5 |
| 44100. | 764.60 | 288.85 | 10.5 |
| 45000. | 726.00 | 288.80 | 10.5 |
| 45900. | 687.40 | 288.75 | 10.5 |
| 46800. | 648.80 | 288.70 | 13.7 |
| 47700. | 646.20 | 288.70 | 13.7 |
| 48600. | 643.60 | 288.70 | 13.7 |
| 49500. | 641.00 | 288.70 | 13.7 |
| 50400. | 638.40 | 288.70 | 7.2 |
| 51300. | 594.40 | 288.70 | 7.2 |
| 52200. | 550.40 | 288.67 | 7.2 |
| 53100. | 506.40 | 288.65 | 7.2 |

| | | | |
|---|---|---|---|
| 54000 | 462.40 | 288.62 | 6.0 |
| 54900. | 419.60 | 288.60 | 6.0 |
| 55800. | 376.80 | 288.57 | 6.0 |
| 56700. | 334.00 | 288.55 | 6.0 |
| 57600. | 291.20 | 288.52 | 5.5 |
| 58500. | 250.80 | 288.50 | 5.5 |
| 59400. | 210.40 | 288.32 | 5.5. |
| 60300. | 170.00 | 288.15 | 5.5 |
| 61200. | 129.60 | 287.97 | 11.0 |
| 62100. | 102.80 | 287.80 | 11.0 |
| 63000. | 76.00 | 287.62 | 11.0 |
| 63900. | 49.20 | 287.45 | 11.0 |
| 64800. | 22.40 | 287.27 | 18.2 |
| 65700. | 0.0 | 287.10 | 18.2 |
| 66600. | 0.0 | 286.98 | 18.2 |
| 67500. | 0.0 | 286.85 | 18.2 |
| 68400. | 0.0 | 286.73 | 30.4 |
| 69300. | 0.0 | 286.60 | 30.4 |
| 70200. | 0.0 | 286.45 | 30.4 |
| 71100. | 0.0 | 286.30 | 30.4 |
| 72000. | 0.0 | 286.15 | 25.0 |
| 72900. | 0.0 | 286.00 | 25.0 |
| 73800. | 0.0 | 285.85 | 25.0 |
| 74700. | 0.0 | 285.70 | 25.0 |
| 75600. | 0.0 | 285.55 | 18.0 |
| 76500. | 0.0 | 285.40 | 18.0 |
| 77400. | 0.0 | 285.10 | 18.0 |
| 78300. | 0.0 | 284.80 | 18.0 |
| 79200. | 0.0 | 284.50 | 14.0 |
| 80100. | 0.0 | 284.20 | 14.0 |
| 81000. | 0.0 | 284.10 | 14.0 |
| 81900. | 0.0 | 284.00 | 14.0 |
| 82800. | 0.0 | 283.90 | 13.0 |
| 83700. | 0.0 | 283.80 | 13.0 |
| 84600. | 0.0 | 283.45 | 13.0 |
| 85500. | 0.0 | 283.10 | 13.0 |
| 86400. | 0.0 | 282.4 | 13.0 |

```
C-----------------------------------------------------
C FOURTH COLUMN FOR VITRO AND ASHRAE PROFILES ARE GIVEN BELOW (KG/S
C-----------------------------------------------------
  VITRO    ASHRAE
C---------------------------
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
```

| | |
|---|---|
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 46.0 | 0.0 |
| 46.0 | 0.0 |
| 46.0 | 0.0 |
| 46.0 | 0.0 |
| 109.7 | 89.67 |
| 109.7 | 89.67 |
| 109.7 | 89.67 |
| 109.7 | 89.67 |
| 19.0 | 0.0 |
| 19.0 | 0.0 |
| 19.0 | 0.0 |
| 19.0 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 12.0 | 89.67 |
| 12.0 | 89.67 |
| 12.0 | 89.67 |
| 12.0 | 89.67 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 12.0 | 0.0 |
| 12.0 | 0.0 |
| 12.0 | 0.0 |
| 12.0 | 0.0 |
| 19.0 | 89.67 |
| 19.0 | 89.67 |

| | |
|---|---|
| 19.0 | 89.67 |
| 19.0 | 89.67 |
| 12.0 | 0.0 |
| 12.0 | 0.0 |
| 12.0 | 0.0 |
| 12.0 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 8.8 | 0.0 |
| 4.12 | 0.0 |
| 4.12 | 0.0 |
| 4.12 | 0.0 |
| 4.12 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |

# APPENDIX B

## Simulation program for System#2

```
C-----------------------------------------------------------
C 11/6/89
C-----------------------------------------------------------
C THIS PROGRAM IS TAILORED TO REPRESENT THE SYSTEM #2
C IFLAG IS USED TO IDENTIFY ITERATION NO
C NOTE: IN THIS PROGRAM UNIT OF TIME IS SECS
C Y1 ---- STORAGE TANK TEMPERATURE (DEGREE KELVIN)
C YP ---- PLATE TEMPERATURE (DEGREE KELVIN)
C TC ---- COLLECTOR TEMPERATURE (DEGREE KELVIN)
C DELT --  TIME STEP IN SECS
C AC  ---  COLLECTOR AREA
C MC  ---  COLLECTOR FLOW RATE
C V   ---  STORAGE TANK VOLUME
C FR = 0.88 (APPROXIMATELY)
C As = SURFACE AREA OF THE STORAGE TANK
C Qs = SOLAR RADIATION
C Ta = AMBIENT TEMPERATURE (K)
C Ms = MASS OF STORAGE TANK (KG)
C UO(J)--  LOAD FLOW RATE (LITERS/S)
C SUMMATION OF UO = M (TOTAL LOAD FLOW)
C WE  ---  AUXILIARY HEAT TO BE ADDED TO MEET THE LOAD
C SC  ---  ENTROPY GENERATION IN THE COLLECTOR
C SS  ---  ENTROPY GENERATION IN STORAGE TANK
C SM  ---  ENTROPY GENERATION DUE TO MIXING
C-----------------------------------------------------------
      IMPLICIT REAL *8 (A-H,O-Z)
      REAL *8 MS, MC
      DIMENSION TA(100),QS(100),QS1(100),TC(100),UO(100),Y1(100),
     & SC(100),SS(100),TT(100),WE(100),UT(100),SM(100),SW(100),
     & YP(100),UTN(100),STOT(100),STT(100)
C-----------------------------------------------------------
      AC=5.00
      CP=4190.
      CS=4190.
      UL=4.
      US=0.5
C     V = 250.
C     V = 350.
      V = 450.
C----------------
      AS=6.0*(V/1000.)**(2./3.)
C----------------
      MS=V
      TL = 273.+20.
      TM = 273.+20.
      TD = 273.+60.
      Y1(1) = 333.
      DELT = 900.
      MC=0.10
      FR1=MC*CP/(AC*UL)
      FR = FR1*(1.-DEXP(-0.9/FR1))
```

```
C--------------------------------------------------------
      WRITE(6,988)
 988  FORMAT(4X,'RAND PROFILE'//)
      WRITE(6,989)
 989  FORMAT(4X,'VS=250.     VS/AC = 50.      M/VS =1.0')
C--------------------------------------------------------
      DO 11 J = 1,97
      READ(5,*)TT(J),QS(J),TA(J),UO(J)
C----------
C     UO(J) = 0.5*(UO(J)/3600.)*(250./269.)
C     UO(J) = 1.0*(UO(J)/3600.)*(250./269.)
C     UO(J) = 1.5*(UO(J)/3600.)*(250./269.)
C--------------
C     UO(J) = 0.5*(UO(J)/3600.)*(350./269.)
C     UO(J) = 1.0*(UO(J)/3600.)*(350./269.)
C     UO(J) = 1.5*(UO(J)/3600.)*(350./269.)
C----------------
C     UO(J) = 0.5*(UO(J)/3600.)*(450./269.)
C     UO(J) = 1.0*(UO(J)/3600.)*(450./269.)
      UO(J) = 1.5*(UO(J)/3600.)*(450./269.)
C--------------------------------------------------------
  11  CONTINUE
C--------------------------------------------------------
      QS1(1) = QS(1) -UL*(Y1(1)-TA(1))
      IF(QS1(1) .LE. 0.0)QS1(1)=0.0
      WETOT = 0.0
      WRITE(6,99)
  99  FORMAT(//2X,'TIME',10X, 'Y1',13X,'QS1',14X,'WE',14X,'WETOT'//)
C--------------------------------------------------------
      DO 10 J = 2,97
      UT(J-1) = UO(J-1)
      IF(Y1(J-1) .GT. 333.) WE(J-1)=0.0
   6  Y1(J) = Y1(J-1)+ (DELT/(MS*CS))*( AC*FR*QS1(J-1)
     &      - AS*US*(Y1(J-1)-TA(J-1)) - UT(J-1)*CP*(Y1(J-1)-TL) )
      IF( (UO(J-1) .GT. 0.0) .AND. (Y1(J-1) .LT. 333.) )THEN
      GOTO 5
      ELSEIF( (UO(J-1) .GT. 0.0) .AND. (Y1(J-1) .GE. 333.) )THEN
      GOTO 15
      ENDIF
C--------------------------------------------------------
C     IF Y1(J-1) IS LESS THAN 333. ADD MORE WE
C     TO INCREASE Y1(J-1) TO 333.
C--------------------------------------------------------
   5  QS1(J) = QS(J)-UL*(Y1(J)-TA(J))
      IF(QS1(J) .LE. 0.0)QS1(J)=0.0
      WE(J-1) = UT(J-1)*CP*(333.-Y1(J-1))
      IF(WE(J-1) .LE. 0.0) WE(J-1) = 0.0
      WETOT = WETOT + WE(J-1)
      WRITE(6,33)(J-1)*0.25,Y1(J),QS1(J),WE(J-1),WETOT
  33  FORMAT(2X,F5.2,4E16.5)
C--------------------------------------------------------
C     WRITE(6,33)(J-1)*0.25,Y1(J),WE(J-1)*0.9,WETOT*0.9
C 33  FORMAT(2X,F5.2,4E16.5)
C     WRITE(6,33)(J-1)*0.25,Y1(J)
C 33  FORMAT(2X,F5.2,E16.5)
C--------------------------------------------------------
```

```
      SW(J) = UT(J-1)*CP*DLOG(333./Y1(J-1))
      SM(J) = 0.0
      GOTO 10
C-----------------------------------------------------------
C     IF Y1(J-1) IS GREATER THAN 333. MIXING IS DONE
C     TO REDUCE Y1(J-1) TO 333.
C-----------------------------------------------------------
15    UT(J-1) = UO(J-1)*(333.-293.)/(Y1(J-1)-293.)
      SM(J) = UT(J-1)*CP*DLOG(333./Y1(J-1))
     &      + (UO(J-1)-UT(J-1))*CP*DLOG(333./TL)
      SW(J) = 0.0
C-----------------------------------------------------------
      QS1(J) = QS(J)-UL*(Y1(J)-TA(J))
      IF(QS1(J) .LE. 0.0) QS1(J)=0.0
      WE(J-1) = 0.0
      IF(Y1(J-1) .GT. 333.) WE(J-1)=0.0
      WETOT = WETOT + WE(J-1)
      WRITE(6,77)(J-1)*0.25,Y1(J),QS1(J),WE(J-1),WETOT
   77 FORMAT(2X,F5.2,4E12.5)
C-----------------------------------------------------------
C     WRITE(6,77)(J-1)*0.25,Y1(J), WE(J-1)*0.9, WETOT*.9
C  77 FORMAT(2X,F5.2,4E12.5)
C     WRITE(6,77)(J-1)*0.25,Y1(J)
C  77 FORMAT(2X,F5.2,E12.5)
C-----------------------------------------------------------
10    CONTINUE
C-----------------------------------------------------------
C           CALCULATION OF ENTROPY
C-----------------------------------------------------------
      TOT1 = 0.0
      TOT3 = 0.0
      TOT2 = 0.0
      STT(1)=0.0
C ------------
      DO 20 J = 2, 97
      YP(J-1) = Y1(J-1) + QS1(J-1)*(1.-FR)/UL
      IF(QS1(J-1) .LE. 0.0)THEN
      QS1(J-1) = 0.0
      YP(J-1) = TA(J-1) +QS(J-1)/UL
      ENDIF
      SC1 = DELT*(-QS(J-1)*AC/YP(J-1))
      SC2 = DELT*(+AC*UL*(YP(J-1)-TA(J-1))/TA(J-1))
      SC(J)  = SC1 +SC2
C -------
      SS1 = DELT*(+ AC*FR*QS1(J-1)/Y1(J-1))
      SS2 = DELT*(- US*AS*(Y1(J-1)-TA(J-1))/Y1(J-1))
      SS3 = DELT*(+UT(J-1)*CP*(TL-Y1(J-1))/Y1(J-1))
      SS5 = DELT*(+ AS*US*(Y1(J-1)-TA(J-1))/TA(J-1))
      SS6 = DELT*(+UT(J-1)*CP*DLOG(Y1(J-1)/TL) )
      SS(J)= SS1+SS2+SS3+SS5+SS6
      STOT(J) =SC(J) +SS(J) +DELT*SM(J) +DELT*SW(J)
      STT(J) = STT(J-1)+STOT(J)
      TOT1 = TOT1+SC(J)
      TOT2 = TOT2+SS(J)
      TOT3 = DELT*SM(J) +DELT*SW(J) +SC(J)+ SS(J) +TOT3
20    CONTINUE
```

```
C------------------------------------------------------
C    WRITE(6,55)TOT1,TOT2,TOT3
C------------------------------------------------------
C  FOR GETTING ENTROPY IN KJ  DIVIDE BY 1000. (BECAUSE TOT3 IS IN J
C------------------------------------------------------
C    WRITE(6,55)TOT1,TOT2,TOT3/1000.
C55  FORMAT(/4X,3E15.5)
C    WRITE(6,7)
C 7  FORMAT(//6X,'TIME     Y1     SC     SS    SM    STO'//)
C    DO 30 J = 2,97
C    WRITE(6,44) (J-1)*0.25, Y1(J),SC(J),SS(J),SM(J),STOT(J),STT(J
C 44  FORMAT(2X,F5.2,6E12.4)
C------------------------------------------------------
C FOR GETTING STOT(J) IN KJ  DIVIDE BY 1000. AND SINCE TOT3 IS IN J
C------------------------------------------------------
C    WRITE(6,44) (J-1)*0.25, STOT(J)/1000. , WE(J)*.9/150.
C 44  FORMAT(2X,F5.2,2E12.4)
C 30  CONTINUE
C------------------------------------------------------
      STOP
      END
```

# APPENDIX C
## Dynamic Programming Algorithm

### DYNAMIC PROGRAMMING METHOD:

The purpose of this appendix is to give a detailed exposition of the basic dynamic programming computational procedure [55]. The computational implications of the principle of optimality are made explicit here. The problem to which dynamic programming applies is the optimization of multistage decision processes. The essential elements of the problem are the system equations, which describe the dynamic behavior of the process under consideration, the performance criterion, which evaluates a particular decision policy; and the constraints which place restrictions on the system operations. Moreover for a problem to have physical significance, all state and control variables must have explicit constraints to limit their range. The system equations are a set of relations between three types of variables: the stage variable, the state variable, and the decision variables. They actually describe how the state variables of stage $k+1$ are related to the state variables at stage k and the decision variables at stage k.

$$x(k+1) = g\{x(k), u(k), k\} \tag{C.1}$$

The stage variable is that quantity which determines the order in which events occur in the system. Very often, time is taken as the stage variable and is assumed to take discrete values k, $k = 0,1,2,\ldots.. K$. The state variables are a set of variables that completely describe the system in the sense that if their values are known for all k, $k = 0,1,2,\ldots.. K$, together with the system inputs (decisions), then any question about the behavior of the system for this range of 'k' can be answered. For any given problem a set of 'n' independent variables ($x_1, x_2 ... x_n$) can be specified.

The decision variables are those variables in the process that can be chosen directly. These variables influence the process by affecting the state variables in some prescribed fashion. In general there could be 'm' decision variables ($u_1, u_2 ... u_m$).

The performance criterion provides an evaluation of a given decision sequence, u(0), u(1),.......u(K). The criterion function depends on each value of u(k) in the decision sequence and also on each value of the state vector, x(0), x(1),......x(K).

In pursuit of our goal of obtaining optimal policies, we shall use a basic principle, which Bellman has called "The Principle of Optimality". This key concept spells out that if an optimal sequence of states passes through a particular state 'x' at stage k, then the portion of the sequence from x(k) to the end of the process must be the optimal sequence from x(k) to the end.

**Dynamic Programming Algorithm**

The algorithm minimizes the cost function

$$J = \sum_{k=0}^{K} L\left\{x(k), u(k), k\right\} \tag{C.2}$$

subject to the system differential equation relating the state variables x(k) of stage k+1 to the state variables at stage k and the decision variables u(k) at stage k

$$x(k+1) = g\{x(k), u(k), k\} \tag{C.3}$$

The constraints placing restrictions on the values that the state variables and decision variables can assume are mathematically expressed as

$$x \varepsilon X(k) \tag{C.4}$$

$$u \varepsilon U(x,k) \tag{C.5}$$

The following algorithm may be used:

1. In order to apply to the dynamic programming computational procedure there must be a finite number of admissible states and admissible decisions. This requirement is usually met by quantizing these variables. Within the range determined by Eq.(C.4), each state variable $x_i$ is quantized with uniform increment $\Delta x_i$.

2. The decision variables are also quantized. The set of admissible decisions U, can be denoted as

$$U = \left\{u^{(1)}, u^{(2)}, \ldots\ldots, u^{(M)}\right\} \tag{C.6}$$

   where M is the total number of admissible controls.

3. The stage variable is quantized so that it takes on values 0,1,2,...K, where K is the total number of stages.

4. Specify a set of boundary conditions i.e. values of minimum cost function for every quantized state $x \varepsilon X$ at stage $k = K$

$$I(x,K) = \min \left\{ L(x,u,K) \qquad u \varepsilon U \right\} \tag{C.7}$$

Usually no decision is made at stage $k=K$ therefore $I(x,K)$ can be written directly as

$$I(x,K) = L(x,K) \tag{C.8}$$

5. Knowing $I(x,K)$ for all $x \varepsilon X$, it is possible to compute the optimal decision by iterative application of the functional equation, Eq. (C.8).

$$I(x,k) = \min \left\{ L(x,u,k) + I(g(x,u,k),k+1) \right\} \tag{C.10}$$

where $u \varepsilon U$ and

$k = 0,1,\ldots\ldots K\text{-}1$

6. Consider a quantized state $x \varepsilon X$, at stage (K-1). At this state, each of the admissible decisions $u^{(m)} \varepsilon U$ is applied. For each of these decisions the cost at the current stage can be determined as

$$L^{(m)} = L\left[x,u^{(m)},K\text{-}1\right] \quad (m = 1,2,\ldots\ldots M) \tag{C.10}$$

7. For each of $u^{(m)} \varepsilon U$ decisions the next state at stage K is determined from the system equation, Eq.(C.3)

$$x^{(m)}(K) = g\left[x,u^{(m)},K\text{-}1\right] \quad (m = 1,2,\ldots\ldots M) \tag{C.11}$$

8. The next step is to compute the minimum cost at stage K for each of the states $x^{(m)}$. However, in general a particular state $x^{(m)}$ will not lie on one of the quantized states $x \varepsilon X$ at which the optimal cost $I(x,K)$ is defined. If $x^{(m)}$ lies outside of the range of admissibles states determined

by Eq. (5.3), then no further consideration is given to that state since it violates the constraints.

9. If a next state $x^{(m)}$ does fall within the range of allowable states, but not on a quantized value, then it is necessary to use some interpolation scheme to compute the minimum cost function at these points. Then the values of the minimum cost at the states $x^{(m)}$ can be expressed as a function of the values of the optimal cost at quantized states $x \varepsilon X$.

$$I(x^{(m)},K) = P\left\{x^{(m)}, K, I(x,K)\right\}, \quad (\text{all } x \; \varepsilon \; X) \tag{C.12}$$

A linear interpolation scheme is used in this research as shown in Fig.(C.1)

10. The total cost of applying decision $u^{(m)}$ at state x, stage (K-1), can be written as

$$F_1^{(m)} = L\left[x, u^{(m)}, K\text{-}1\right] + I\left[x^{(m)}, K\right] \tag{C.13}$$

this is the quantity which is to be minimized by choice of $u^{(m)}$ in the functional equation, Eq.(C.9). The minimization can be achieved by simply comparing the M quantities. The minimum value will be the minimum cost at state x, stage (K-1).

$$I\left[x,K\text{-}1\right] = \min\left\{L\left[x, u^{(m)}, K\text{-}1\right] + I\left[x^{(m)}, K\right]\right\} \tag{C.14}$$

$$\text{where} \quad u^{(m)} \varepsilon U$$

The optimal decision at this state and stage, $u\left[x,K\text{-}1\right]$, is the control $u^{(m)}$ for which the minimum in Eq. (C.14) is actually taken on.

11. This procedure is repeated at each quantized state $x \varepsilon X$ at stage (K-1). When this has been done, I(x,K-1) and u(x,K-1) are known for all $x \varepsilon X$. Therefore I(x,K-2) and u(x,K-2) can be computed for all $x \varepsilon X$. based on the knowledge of I(x,K-1). The procedure continues until I(x,0) and u(x,0) have been computed.

The dynamic programming solution is a specification of u(x,k) and I(x,k) for all quantized $x \varepsilon X$ for k = 0,1,2,.......K. However the original problem is to find the optimum sequence of decisions starting from a given x(0) (tracing of optimal trajectory from a given starting point). The first decision in the sequence is evaluated as

$$u(0) = u[x(0),0] \tag{C.15}$$

The next state along the sequence is then found by applying the system difference equation to obtain

$$x(1) = g\{x(0),u(0),0\} \tag{C.16}$$

This state may or may not be a quantized state. If it is, then the next decision in the optimum sequence is evaluated directly as

$$u(1) = u\{x(1),1\} \tag{C.17}$$

If it is not a quantized state, then the value of u(1) must be found by linear interpolation scheme.

The recovery procedure, in which the next state is computed on the basis of the present state and the optimal decision function evaluated at the present state utilizing the function u(x,k), continues until the complete decision sequence u(0), u(1), ....u(K) and optimal trajectory x(0), x(1),......x(K) have been obtained.

Thus dynamic programming involves two sweeps through the stage variable. The first sweep is working backwards, computing I(x,k), the minimum cost function at state x and stage 'k', in terms of I(x,k+1), the minimum cost function at stage k+1. The second sweep is working forward, recovering the optimum decision and optimal trajectory in state space by forward iteration of the system $x(k+1) = g\{x(k),u(k),k\}$ where the optimum decision policy function $u[x(k),k]$ is used to determine the optimum decisions at each stage.

For better understanding of the above procedure, a simple problem has been worked out below and a complete flow chart of the procedure is presented Fig.(C.2).

## SAMPLE PROBLEM

System equation is given as:

$$\dot{x} = x + u \tag{C.18}$$

The performance criterion which is to be minimized is:

$$J = \sum_{k=1}^{11}(x^2 + u^2) \tag{C.19}$$

The state variable is constrained to lie in the interval:

$$0 \le x \le 8 \tag{C.20}$$

while the decision variable is bounded by

$$-2 \leq u \leq 2 \tag{C.21}$$

Initial cost at the boundary condition is given as:

$$I(2,11) = 1.0$$

**Solution:**

The state variable is quantized in uniform increments of one thus

$$X = \{0,1,2,3,4,5,6,7,8,\}$$

is the set of admissible states. Each quantized state is indexed with j where j = 1,2,3,........Jmax, such that

$$\{x(1) = 0, x(2) = 1, ,........x(Jmax) = 8\}$$

The decision variable is also quantized with uniforms increments of 0.5, so that the set of admissible decisions is

$$U = \{-2.0,-1.5,-1.0,-0.5,0.0,0.5,1.0,1.5,2.0\}$$

Each discrete value of u is indexed as m where m = 1,2,3,......Mmax, such that

$$\{u(1) = -2.0, u(2) = -1.5, ,........u(Mmax) = 2.0\}$$

The stage variable is also quantized, each stage is indexed as k, where k = 1,2,3,......Kmax

At k=Kmax initial cost is given as I(3,11)=1; this implies that at k=Kmax, j=3 is the only admissible state. The inadmissible states are identified by assigning to them large costs. These initial costs or setting is useful in initiating the backward optimization process.

**Algorithm** Set the stage $k^- = (Kmax\text{-}1)$. Consider first state $j=1$ at $k=(Kmax\text{-}1)$, at this state every admissible decision $u \varepsilon U$ is applied. For each of these decisions, the next state is computed from dynamic equation Eq.(C.22):

$$xk = x(j) + u(m)$$

The next xk is examined whether it satisfies Eq.(C.20). If the new state is not admissible, then no further consideration is given to this state, since it violates the constraints. The cost associated with the state is assigned a large value (1.0e+20)

If the next state is an admissible one, then the cost at this state is directly evaluated as Ik (Ik is also called next stage minimum cost), and the total cost at current state corresponding j1, is calculated using Eq.(C.25) which is called iterative functional equation.

$$TI(m) = L(m) + IK \tag{C.25}$$

where L(m) is the single stage cost at the current state and is given as

$$L(m) = [x(j)]^2 + [u(j)]^2 \tag{C.26}$$

If the next state xk is an admissible, but it if it lies between two quantized states (i.e., between states j and j+1), then the next stage minimum cost Ik is determined by linear interpolation scheme as shown in Fig.(C.1) and Eq.(C.27)

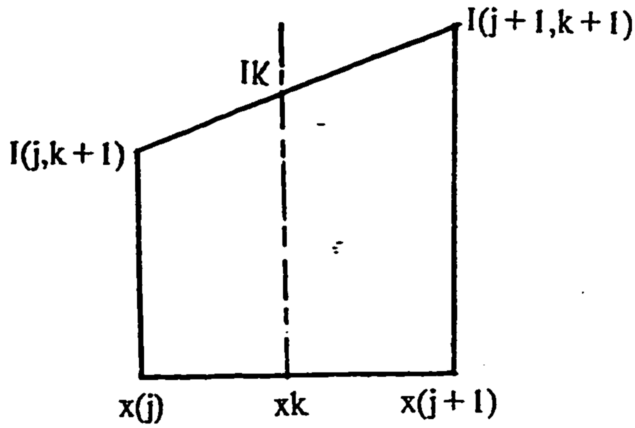



Fig.(C.1) One Dimensional Linear Interpolation

$$IK = I(j,k+1) + \frac{I(j+1,k+1) - I(j,k+1)}{x(j+1) - x(j)}(xk - x(j)) \tag{C.27}$$

The total cost at state xk, stage k=Kmax-1 is given by Eq.(C.25). This is the quantity to be minimized by choice of u(m). Minimization is achieved by comparing all TI(m) values. The minimum will be the minimum cost [I(j,k) = minimum of all TI(m) values] at state xk and stage Kmax-1, the optimal decision umin(j,k) corresponds to the minimum cost indexed by I(j,k).

The procedure is repeated at every quantized state and stage k=10, then the minimum cost I(j,10) and optimal decision umin(j,10) will be known for all quantized states at k=10. The procedure is continued until k=1 is reached.

**Tracing the optimal trajectory from the starting xk=2.**

The optimum sequence from a given initial point xk=2 at stage can be easily determined from the dynamic programming solution. The first decision in the sequence is u(1). The next state is found by applying the system difference equation Eq.(C.18) by using the optimal decision u(1). If the next is a quantized one, then the next optimal decision is evaluated as u(2). If the next state is not quantized then athe optimal decision is found by linear interpolation scheme (as illustrated above).

The recovery procedure is continued until complete decision sequence u(1),u(2),.....u(Mmax) and optimal trajectory x(1),x(2),......x(K) have been obtained.

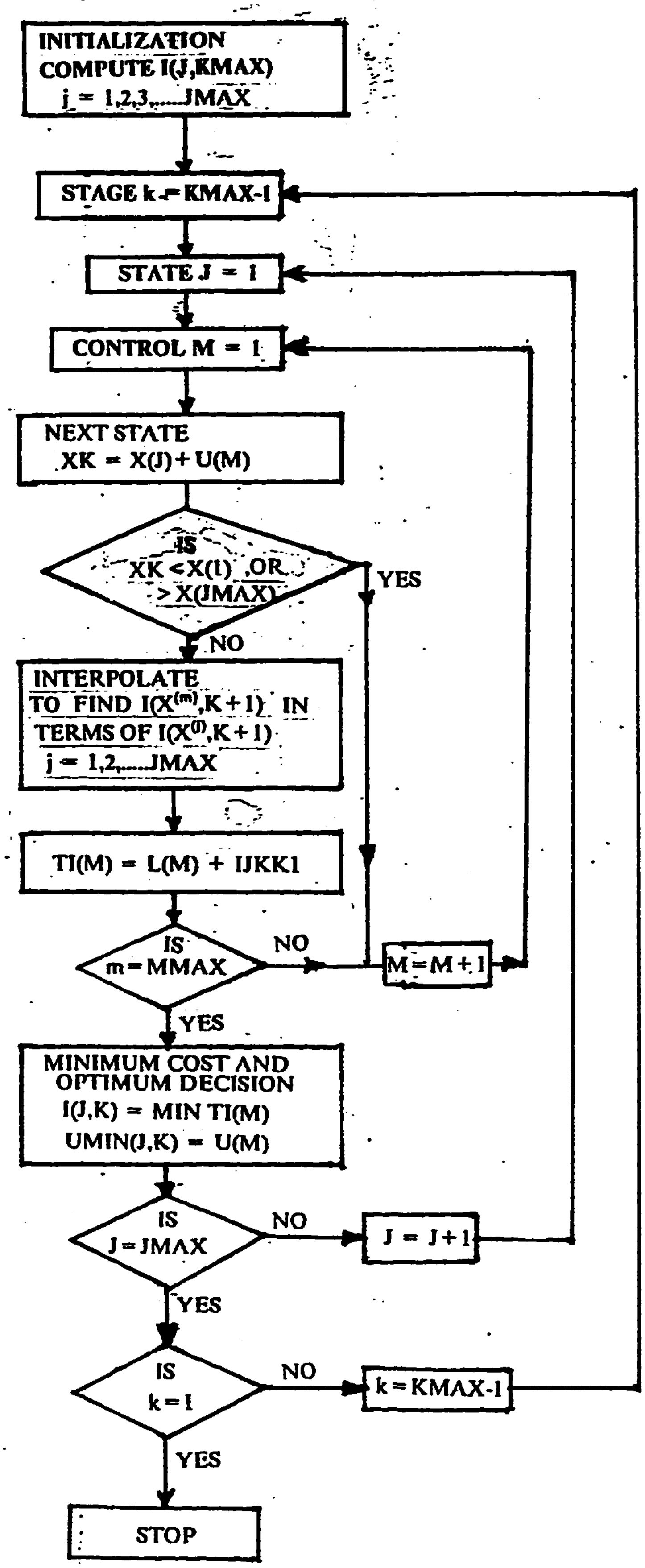



Fig.(C.2) Flow Chart of the Dynamic Programming Sample Problem

```
1
0.400E+01 0.400E+01 0.400E+01 0.400E+01 0.400E+01 0.400E+01 0.400E+01 0.400E+01 0.400E+01 0.500E+01 0.100E+21
    0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000     1.000     2.000     0.000

2
0.600E+01 0.600E+01 0.600E+01 0.600E+01 0.600E+01 0.600E+01 0.575E+01 0.500E+01 0.400E+01 0.300E+01 0.100E+21
   -1.000    -1.000    -1.000    -1.000    -1.000    -1.000    -0.500     0.000     0.000     1.000     0.000

3
0.110E+02 0.110E+02 0.110E+02 0.110E+02 0.110E+02 0.108E+02 0.100E+02 0.900E+01 0.800E+01 0.500E+01 0.100E+01
   -1.000    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000     0.000     0.000

4
0.190E+02 0.190E+02 0.190E+02 0.190E+02 0.190E+02 0.188E+02 0.180E+02 0.170E+02 0.150E+02 0.110E+02 0.100E+21
   -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -1.000    -1.000     0.000

5
0.310E+02 0.310E+02 0.310E+02 0.310E+02 0.308E+02 0.300E+02 0.290E+02 0.280E+02 0.250E+02 0.210E+02 0.100E+21
   -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000     0.000

6
0.480E+02 0.480E+02 0.480E+02 0.480E+02 0.478E+02 0.470E+02 0.460E+02 0.440E+02 0.400E+02 0.100E+21 0.100E+21
   -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000     2.000     0.000

7
0.710E+02 0.710E+02 0.710E+02 0.708E+02 0.700E+02 0.690E+02 0.680E+02 0.650E+02 0.610E+02 0.100E+21 0.100E+21
   -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000     2.000     0.000

8
0.101E+03 0.101E+03 0.101E+03 0.101E+03 0.100E+03 0.990E+02 0.970E+02 0.930E+02 0.100E+21 0.100E+21 0.100E+21
   -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000     2.000     2.000     0.000

9
0.139E+03 0.139E+03 0.139E+03 0.138E+03 0.137E+03 0.136E+03 0.133E+03 0.129E+03 0.100E+21 0.100E+21 0.100E+21
   -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000    -2.000     2.000     2.000     0.000

 TOTAL COST =    14.0000
```

**Output of the Sample Problem Illustrated in Appendix C**

# APPENDIX D

## Optimization program for System#1

```
/*JOBPARM LINES=99
//S1 EXEC FORTVCLG,PARM.FORT='LANGLVL(77)',REGION.FORT=5000K,
// REGION.GO=5000K
//FORT.SYSIN DD *
C-----------------------------------------------------------------
C     27/5/89
C-----------------------------------------------------------------
C IN THIS PROGRAM THERE ARE TWO STATE VARIABLES Y1,Y2
C Y1 ---- STORAGE TANK TEMPERATURE (DEGREE KELVIN)
C Y2 ---- AMOUNT OF WATER TO BE ALLOCATED TO THE REMAINING K STAGES
C K ----  REFERS TO STAGE - TIME
C J ----  REFERS TO STATES
C M1 ----  REFERS TO DIFFERENT FLOW RATE VALUES
C M2 ----  REFERS TO DIFFERENT AUXILIARY ENERGY VALUES
C DELT --- TIME STEP IN SECS
C UO(M1)--- FLOW RATE (LITERS/S)
C Y1K ---  VALUE OF THE STATE VARIABLE AT THE NEXT STAGE
C FR = 0.88 (APPROXIMATELY)
C As = SURFACE AREA OF THE STORAGE TANK
C Qs = SOLAR RADIATION
C Ta = AMBIENT TEMPERATURE (K)
C Ms = MASS OF STORAGE TANK (KG)
C SCK = ENTROPY GENERATION IN THE COLLECTOR
C SSK = ENTROPY GENERATION IN THE STORAGE TANK
C SMK = ENTROPY GENERATION IN THE MIXING VALVE
C-----------------------------------------------------------------
C PART 2 OF THE PROGRAM : TRACING OF OPTIMAL TRAJECTORY
C UUO(K) --- OPTIMAL FLOW RATE AT K STAGE
C WWE(K) --- OPTIMAL AUXILIARY ENERGY AT STAGE K
C LL(K) ---- TOTAL ENTROPY AT THE END OF STAGE K
C I(J1,J2,KMAX), UM(J1,J2,KMAX), WM(J1,J2,KMAX), Y1(J1),Y2(J2),UO(M),
C WE(M2), L(M1,M2), TI(M1,M2),UT(M1),UTN(M1),YP(J1)
C SC(J1,J2),SS(J1,J2),SM(J1,J2)
C UUO(K),WWE(K),UUT(K)
C-----------------------------------------------------------------
      IMPLICIT REAL*8(A-H, O-Z)
      REAL *8 MS, MC, I(41,51,97), UM(41,51,97), WM(41,51,97),
     &    L(51,41), TI(51,41), IJKK1, IJKK2, IJK,IK, LL(97)
      DIMENSION TT(97), QS(97), TA(97), QS1(97),
     &      Y1(41), Y2(51), UO(51), WE(41),
     &      SC(41,51), SS(41,51), SM(41,51),
     &      UT(51), YP(41), UTN(51),
     &      UUO(97), WWE(97), UUT(97)
C-----------------------------------------------------------------
C           READING AND INITIALIZATION
C-----------------------------------------------------------------
      KMAX = 97
      J1MAX = 41
      J2MAX = 51
      M1MAX = 51
      M2MAX = 41
```

```
C-----------------------
      DO 1 J1 = 1,J1MAX
      CJ1 = J1
    1 Y1(J1) = 330. + (CJ1-1.)
C-----------------------
      DO 2 J2 = 1,J2MAX
      CJ2 = J2
    2 Y2(J2) = (CJ2 -1.0)*13.5
C-----------------------
      DO 3 M1 = 1,M1MAX
      CM1 = M1
    3 UO(M1) = ( (CM1 -1.0)*54.0)/3600.
C-----------------------
      DO 4 M2 = 1,M2MAX
      CM2 = M2
    4 WE(M2) = (CM2 -1.0)*300.
C-----------------------
      DO 5 J1 = 1,J1MAX
      DO 5 J2 = 1,J2MAX
      UM(J1,J2,97) = 0.0
      WM(J1,J2,97) = 0.0
    5 CONTINUE
C-----------------------
      DO 6 K  = 1,KMAX
      DO 6 J1 = 1,J1MAX
      DO 6 J2 = 1,J2MAX
    6 I(J1,J2,K) = 1.0E+50
C-------------------------------------------------------
C INITIALIZATION OF COST AT THE BOUNDARY CONDITIONS
C FOR ANY CHANGE IN J1 &J2 VALUES, CHANGE THE SUBSCRIPTS OF I
C FOR ANY CHANGE IN TERMINAL TS, CHANGE THE SUBSCRIPTS  OF I
C-------------------------------------------------------
      I(2,50,97) = 1.0E+10
      I(2,51,97) = 1.0E+10
      I(3,50,97) = 1.0E+5
      I(3,51,97) = 1.0E+5
      I(4,50,97) = 1.0E+5
      I(4,51,97) = 1.0
      I(5,50,97) = 1.0E+5
      I(5,51,97) = 1.0E+5
      I(6,50,97) = 1.0E+10
      I(6,51,97) = 1.0E+10
C-----------------------
      DO 7 K = 1,97
      READ(5,*)TT(K),QS(K),TA(K)
    7 CONTINUE
C-----------------------
      AC=5.00
      CP=4190.
      MS=450.
C----------
      CS=4190.
      UL=4.
      US=0.5
      AS=3.52
C----------
```

```
      TL = 273.+20.
      TM = 273.+20.
      TD = 273.+60.
      DELT = 900.
C------------------------------
C  FR FOR CONSTANT COLLECTOR FLOW RATE
C------------------------------
C    MC=0.10
C    FR1=MC*CP/(AC*UL)
C    FR = FR1*(1.-DEXP(-0.9/FR1))
C-------------------------------------------------
      DO 10 K = KMAX-1, 1, -1
       DO 15 J1 = 1, J1MAX
         QS1(K) = QS(K) -UL*(Y1(J1)-TA(K))
         IF(QS1(K) .LE. 0.0) THEN
         QS1(K) = 0.0
         MC = 0.
         FR = 0.01
         GOTO 602
         ELSEIF(QS1(K) .GT. 0.0)THEN
C-------------------------------------------------
C      EXPRESSION FOR OPTIMAL MASS FLOW RATE THROUGH THE COLLECTOR
C-------------------------------------------------
      FP   = 0.9
      AL    = 20.0
      D    = 0.01
      RHO  = 1000.0
      F    = 0.03
      ACC   = 2.5
      CP   = 4190.
      CF   =8*F*AL/( (RHO**2)*(3.14**2)*D**5)
      MC =2.* ( (ACC**2*UL*FP**2*QS1(K)*(Y1(J1)-TA(K)))
     &               /(6.0*CF*Y1(J1)*CP) )**0.25
      FR1=MC*CP/(AC*UL)
      FR = FR1*(1.-DEXP(-0.9/FR1))
      ENDIF
C-------------------------------------------------
C    RECTANGULAR GRID IS CONVERTED IN A TRIANGULAR ONE SO AS TO
C    CARRY OUT OPTIMIZATION DURING SOLAR INSOLATION.
C    THIS REDUCTION WAS DONE SINCE AFTER MANY OPTIMIZATION RUNS
C    IT WAS FOUND THAT ENTROPY GENERATION CONCENTRATION WAS HIGH
C    DURING SOLAR INSOLATION PERIOD
C-------------------------------------------------
 602 IF(K .GE. 60)THEN
      J2UP = J2MAX
      J2DN = J2MAX - 6
      ELSEIF(K.LT.60 .AND. K .GE. 48)THEN
      J2UP = J2MAX
      J2DN = J2DN-4
      IF(J2DN .LE. 1)J2DN = 1
      ELSEIF(K .LT. 48 .AND. K .GE. 36)THEN
      J2UP = J2UP - 4
      J2DN = 1
      IF(J2UP .LE. 5) J2UP = 5
      ELSEIF(K .LT. 36)THEN
      J2UP = 5
```

```
      J2DN = 1
      ENDIF
C-----------------------------------------------------
C L(M1) -- SINGLE STATE COST FUNCTION AT A GIVEN POINT
C TI(M1) -- TOTAL COST (ENTROPY) AT A GIVEN POINT
C-----------------------------------------------------
       DO 20 J2 = J2DN, J2UP
C---------
      DO 9 M1=1,M1MAX
      DO 9 M2=1,M2MAX
  9  TI(M1,M2) = 1.0E+50
C----------
        DO 40 M1 = 1,M1MAX
        DO 30 M2 = 1, M2MAX
      UT(M1) = UO(M1)
      IFLAG = 1
 41  Y1K = Y1(J1) + (DELT/(MS*CS))*( AC*FR*QS1(K)
     &         - AS*US*(Y1(J1)-TA(K)) - UT(M1)*CP*(Y1(J1)-TL)
     &         + WE(M2) )
      Y2K = Y2(J2) + UO(M1)*DELT
      IF(Y1K .LT. 333. .AND. UO(M1) .GT. 0.0 )GOTO 30
      IF( Y1K.LT.Y1(1) ) GOTO 30
      IF( Y1K.GT.Y1(J1MAX) ) GOTO 40
      IF( (Y2K.LT.Y2(1)) .OR. (Y2K.GT.Y2(J2UP)) ) GOTO 601
      IF((Y1K-333.) .GT. 0.01 .AND. UO(M1) .GT. 0.) GOTO 42
      GOTO 43
C------------------------------
C       MIXING --DUE TO INCREASE IN TANK TEMP BEYOND 273.+60.
C       SM(J)---> MIXING ENTROPY
C-----------------------------------------------------
 42  UT(M1) = UO(M1)*(333.-293.)/(Y1K-293.)
      IF(IFLAG .EQ. 1)THEN
      UTN(M1) = UT(M1)
      IFLAG = 2
      GOTO 41
      ENDIF
      IF(ABS(UTN(M1)-UT(M1)).GT. (ABS(UT(M1))/20) )THEN
      UTN(M1) = UT(M1)
      GOTO 41
      ENDIF
C-----------------------------------------------------
C  INTERPOLATION
C-----------------------------------------------------
      DO 45 JJ2 = 1,J2MAX-1
      IF( (Y2K .GE. Y2(JJ2)) .AND. (Y2K .LE. Y2(JJ2+1)) ) THEN
      DO 46 JJ1 = 1,J1MAX-1
      IF( (Y1K .GE. Y1(JJ1)) .AND. (Y1K .LE. Y1(JJ1+1)) ) THEN
      JK1 = JJ1
      JK2 = JJ2
C-----------------------------------------------------
C  FOR TWO DIMENSIONAL INTERPOLATION USE THE STATEMENTS IN COMMENT MODE
C-----------------------------------------------------
C   IJKK1 = I(JK1,JK2,K+1) + ( I(JK1,JK2+1,K+1)-I(JK1,JK2,K+1) )*
C  &         (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
C   IJKK2=I(JK1+1,JK2,K+1) +( I(JK1+1,JK2+1,K+1)-I(JK1+1,JK2,K+1) )*
C  &         (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
```

```
C   IJK =IJKK1+(IJKK2-IJKK1)*( Y1K-Y1(JJ1) )/
C  &    (Y1(JJ1+1)-Y1(JJ1))
C------------------------------------------------------------
      IJK = I(JK1,JK2,K+1) + ( I(JK1+1,JK2,K+1)-I(JK1,JK2,K+1) )*
     &          (Y1K-Y1(JJ1))/(Y1(JJ1+1)-Y1(JJ1))
      GOTO 200
      ENDIF
   46 CONTINUE
      ENDIF
   45 CONTINUE
C-----------------
  200 SM(J1,J2) = DELT* ( UT(M1)*CP*DLOG(333./Y1K)
     &      + (UO(M1)-UT(M1))*CP*DLOG(333./TL) )
      GOTO 100
C-----------------
   43 DO 47 JJ2 = 1,J2MAX-1
      IF( (Y2K .GE. Y2(JJ2)) .AND. (Y2K .LE. Y2(JJ2+1)) ) THEN
      DO 48 JJ1 = 1,J1MAX-1
      IF( (Y1K .GE. Y1(JJ1)) .AND. (Y1K .LE. Y1(JJ1+1)) ) THEN
      JK1 = JJ1
      JK2 = JJ2
C------------------------------------------------------------
C   FOR TWO DIMENSIONAL INTERPOLATION USE THE STATEMENTS IN COMMENT MODE
C------------------------------------------------------------
C   IJKK1 = I(JK1,JK2,K+1) + ( I(JK1,JK2+1,K+1)-I(JK1,JK2,K+1) )*
C  &          (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
C   IJKK2=I(JK1+1,JK2,K+1) +( I(JK1+1,JK2+1,K+1)-I(JK1+1,JK2,K+1) )*
C  &          (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
C   IJK =IJKK1+(IJKK2-IJKK1)*( Y1K-Y1(JJ1) )/
C  &    (Y1(JJ1+1)-Y1(JJ1))
C------------------------------------------------------------
      IJK = I(JK1,JK2,K+1) + ( I(JK1+1,JK2,K+1)-I(JK1,JK2,K+1) )*
     &          (Y1K-Y1(JJ1))/(Y1(JJ1+1)-Y1(JJ1))
      GOTO 300
      ENDIF
   48 CONTINUE
      ENDIF
   47 CONTINUE
  300 SM(J1,J2) = 0.0
C------------------------------------------------------------
C       ENTROPY (COST FUNCTION)
C------------------------------------------------------------
  100 IF(QS1(K) .LE. 0.0)THEN
      YP(J1) = TA(K) +QS(K)/UL
      ELSEIF(QS1(K) .GT. 0.0) THEN
      YP(J1) = Y1(J1) + QS1(K)*(1.-FR)/UL
      ENDIF
      SC1 = DELT*(-QS(K)*AC/YP(J1))
      SC2 = DELT*(+AC*UL*(YP(J1)-TA(K))/TA(K))
      SC(J1,J2)  = SC1 +SC2
C------------------------------------------------------------
      SS1 = DELT*(+ AC*FR*QS1(K)/Y1(J1))
      SS2 = DELT*(- US*AS*(Y1(J1)-TA(K))/Y1(J1))
      SS3 = DELT*(+UT(M1)*CP*(TL-Y1(J1))/Y1(J1))
      SS4 = DELT*(+ WE(M2)/Y1(J1))
      SS5 = DELT*(+ AS*US*(Y1(J1)-TA(K))/TA(K))
```

```
      SS6 = DELT*(+UT(M1)*CP*DLOG(Y1(J1)/TL))
      SS(J1,J2) = SS1+SS2+SS3+SS4+SS5+SS6
      L(M1,M2) = SC(J1,J2) +SS(J1,J2) +SM(J1,J2)
      TI(M1,M2) = L(M1,M2) + IJK
   30 CONTINUE
   40 CONTINUE
C-----------------------------------------------------------------
C     FINDING MINIMUM ENTROPY AND ASSOCIATED OPTIMUM UO AND WE
C-----------------------------------------------------------------
  601 I(J1,J2,K) = 1.0E+50
      UM(J1,J2,K)= 0.0
      WM(J1,J2,K)=0.0
      DO 33 M1 = 1,M1MAX
      DO 33 M2 = 1,M2MAX
      IF(TI(M1,M2) .LT. I(J1,J2,K)) THEN
      I(J1,J2,K)=TI(M1,M2)
      UM(J1,J2,K)= UO(M1)
      WM(J1,J2,K)= WE(M2)
      ENDIF
   33 CONTINUE
C-------------
   20 CONTINUE
   15 CONTINUE
   10 CONTINUE
C-----------------------------------------------------------------
C      PRINTING OF BACKWARD OPTIMUM RESULTS
C        (DYNAMIC PROGRAMMING SOLUTION)
C-----------------------------------------------------------------
      DO 49 K = KMAX,1,-1
      IF(K .EQ. 1 .OR. K .EQ. 96)THEN
      WRITE(6,900)K
  900 FORMAT('    K = ',I2)
      WRITE(6,910)(J1,J1=1,10)
  910 FORMAT(10X,10I11)
      DO 50 J2 = 1,J2MAX
      WRITE(6,920)Y2(J2),(I(J1,J2,K),J1=1,10)
  920 FORMAT(11E11.4)
      WRITE(6,922)(3600.*UM(J1,J2,K),J1=1,10)
  922 FORMAT(11X,10E11.4)
      WRITE(6,924)(WM(J1,J2,K),J1=1,10)
  924 FORMAT(11X,10E11.4)
      WRITE(6,925)
  925 FORMAT(' ')
   50 CONTINUE
C-----
      WRITE(6,911)(J1,J1=11,20)
  911 FORMAT(10X,10I11)
      DO 51 J2 = 1,J2MAX
      WRITE(6,912)Y2(J2),(I(J1,J2,K),J1=11,20)
  912 FORMAT(11E11.4)
      WRITE(6,913)(3600.*UM(J1,J2,K),J1=11,20)
  913 FORMAT(11X,10E11.4)
      WRITE(6,914)(WM(J1,J2,K),J1=11,20)
  914 FORMAT(11X,10E11.4)
      WRITE(6,930)
  930 FORMAT(' ')
```

```
 51 CONTINUE
C-----
      WRITE(6,915)(J1,J1=21,31)
 915 FORMAT(10X,11I11)
      DO 52 J2 = 1,J2MAX
      WRITE(6,916)Y2(J2),(I(J1,J2,K),J1=21,31)
 916 FORMAT(12E11.4)
      WRITE(6,917)(3600.*UM(J1,J2,K),J1=21,31)
 917 FORMAT(11X,11E11.4)
      WRITE(6,918)(WM(J1,J2,K),J1=21,31)
 918 FORMAT(11X,11E11.4)
      WRITE(6,935)
 935 FORMAT(' ')
 52  CONTINUE
      ENDIF
 49  CONTINUE
C-----------------------------------------------------
C    TRACING OPTIMAL TRAJECTORY AND MINIMUM ENTROPY
C-----------------------------------------------------
C    INITIALIZATION
C----------------
      WRITE(6,888)
 888 FORMAT(//8X,' Y1K     Y2K     UUT     WWE     IK     WK'//)
      Y1K = 333.
      Y2K = 0.0
      IK = 0.0
      WK = 0.0
C-----------------------------------------------------
      DO 71 K = 1,KMAX-1
C    IF(Y2K.LT.Y2(1)) Y2K=Y2(1)
C    IF(Y2K.GT.Y2(J2MAX)) Y2K=Y2(J2MAX)
C    IF(Y1K.LT.Y1(1)) Y1K=Y1(1)
C    IF(Y1K.GT.Y1(J1MAX)) Y1K=Y1(J1MAX)
      DO 72 JJ2 = 1,J2MAX-1
      IF( (Y2K .GE. Y2(JJ2)) .AND. (Y2K .LE. Y2(JJ2+1)) ) THEN
      DO 73 JJ1 = 1,J1MAX-1
      IF( (Y1K .GE. Y1(JJ1)) .AND. (Y1K .LE. Y1(JJ1+1)) ) THEN
      JK1 = JJ1
      JK2 = JJ2
C-----------------------------------------------------
C    FOR TWO DIMENSIONAL INTERPOLATION
C-----------------------------------------------------
C    UJKK1 =UM(JK1,JK2,K)+(UM(JK1,JK2+1,K)-UM(JK1,JK2,K) )*
C  &    (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
C    UJKK2 = UM(JK1+1,JK2,K)+(UM(JK1+1,JK2+1,K)-UM(JK1+1,JK2,K))*
C  &    (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
C    UUO(K) = UJKK1 + (UJKK2-UJKK1)*(Y1K-Y1(JJ1))/
C  &    (Y1(JJ1+1)-Y1(JJ1))
C    WJKK1 = WM(JK1,JK2,K)+(WM(JK1,JK2+1,K)-WM(JK1,JK2,K) )*
C  &    (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
C    WJKK2 = WM(JK1+1,JK2,K)+(WM(JK1+1,JK2+1,K)-WM(JK1+1,JK2,K))*
C  &    (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
C    WWE(K) = WJKK1 + (WJKK2-WJKK1)*(Y1K-Y1(JJ1))/
C  &    ( Y1(JJ1+1)-Y1(JJ1) )
C-----------------------------------------------------
C    FOR ONE DIMENSIONAL INTERPOLATION
```

```
C----------------------------------------------------------------
      UUO(K) = UM(JK1,JK2,K)+(UM(JK1+1,JK2,K)-UM(JK1,JK2,K))*
     &   (Y1K-Y1(JJ1))/(Y1(JJ1+1)-Y1(JJ1))
      WWE(K) = WM(JK1,JK2,K)+(WM(JK1+1,JK2,K)-WM(JK1,JK2,K))*
     &   (Y1K-Y1(JJ1))/(Y1(JJ1+1)-Y1(JJ1))
      GOTO 74
      ENDIF
 73  CONTINUE
      ENDIF
 72  CONTINUE
 74  UUT(K) = UUO(K)
      IF( (Y1K-333.) .GT. 0.01) GOTO 75
      GOTO 76
 75  UUT(K) = UUO(K)*(333.-293.)/(Y1K-293.)
      SMK= DELT* (UUT(K)*CP*DLOG(333./Y1K)
     &       + (UUO(K)-UUT(K))*CP*DLOG(333./TL) )
      GOTO 500
 76  SMK=0.0
500  QS1(K) = QS(K) -UL*(Y1K-TA(K))
C----------------------------------------------------------------
C       EXPRESSION FOR OPTIMAL MASS FLOW RATE THROUGH THE COLLECTOR
C----------------------------------------------------------------
      IF(QS1(K) .LE. 0.0)THEN
      YPK = TA(K) +QS(K)/UL
      QS1(K) = 0.0
      MC = 0.
      FR = 0.01
      GOTO 603
      ELSEIF(QS1(K) .GT. 0.0)THEN
      FP   = 0.9
      AL   = 20.0
      D    = 0.01
      RHO  = 1000.0
      F    = 0.03
      ACC  = 2.5
      CP   = 4190.
      CF   =8*F*AL/( (RHO**2)*(3.14**2)*D**5 )
      MC =2.* ( (ACC**2*UL*FP**2*QS1(K)*(Y1K-TA(K)))
     &             /(6.0*CF*Y1K*CP) )**0.25
      FR1=MC*CP/(AC*UL)
      FR = FR1*(1.-DEXP(-0.9/FR1))
C----------------------------------------------------------------
      YPK = Y1K + QS1(K)*(1.-FR)/UL
      ENDIF
C---------
603  SC1 = DELT*(-QS(K)*AC/YPK)
      SC2 = DELT*(+AC*UL*(YPK-TA(K))/TA(K))
      SCK  = SC1 +SC2
C-------------------------
      SS1 = DELT*(+ AC*FR*QS1(K)/Y1K)
      SS2 = DELT*(- US*AS*(Y1K-TA(K))/Y1K)
      SS3 = DELT*(+UUT(K)*CP*(TL-Y1K)/Y1K)
      SS4 = DELT*(+ WWE(K)/Y1K)
      SS5 = DELT*(+ AS*US*(Y1K-TA(K))/TA(K))
      SS6 = DELT*(+UUT(K)*CP*DLOG(Y1K/TL) )
      SSK = SS1+SS2+SS3+SS4+SS5+SS6
```

```
      LL(K) = SCK +SSK +SMK
      IK = IK + LL(K)
      WK = WK + WWE(K)
      WRITE(6,899)K,Y1K,Y2K,3600*UUO(K),WWE(K),IK,WK
  899 FORMAT(2X,I4,6E11.4)
      Y1K =Y1K+(DELT/(MS*CS))*( AC*FR*QS1(K)
     &      - AS*US*(Y1K-TA(K))-UUT(K)*CP*(Y1K-TL)+WWE(K) )
      Y2K = Y2K +UUO(K)*DELT
C     IF(K.EQ.97)THEN
      IF(K.EQ.96)THEN
      WRITE(6,950)Y1K,Y2K
  950 FORMAT(5X,2E12.4)
      ENDIF
  71  CONTINUE
      STOP
      END
/*
//GO.SYSIN DD *
C-----------------------------------------------------
C THE DATA USED IN RUNNING THE ABOVE PROGRAM IS GIVEN BELOW
C FIRST COLUMN REPRESENTS TIME IN SECONDS
C SECOND COLUMN REPRESENTS THE SOLAR RADIATION AT A PARTICULAR TIME
C THIRD COLUMN REPRESENTS THE AMBIENT TEMPERATURE AT A PARTICULAR TIME
C-----------------------------------------------------
    0.0     0.0    282.40
   900.     0.0    282.47
  1800.     0.0    282.55
  2700.     0.0    282.62
  3600.     0.0    282.70
  4500.     0.0    282.72
  5400.     0.0    282.75
  6300.     0.0    282.77
  7200.     0.0    282.80
  8100.     0.0    282.85
  9000.     0.0    282.90
  9900.     0.0    282.95
 10800.     0.0    283.00
 11700.     0.0    283.05
 12600.     0.0    283.10
 13500.     0.0    283.15
 14400.     0.0    283.20
 15300.     0.0    283.30
 16200.     0.0    283.40
 17100.     0.0    283.50
 18000.     0.0    283.60
 18900.     0.0    283.65
 19800.     0.0    283.70
 20700.     0.0    283.75
 21600.     0.0    283.80
 22500.     0.0    283.87
 23400.     0.0    283.95
 24300.     0.0    284.02
 25200.    65.68   284.10
 26100.   108.26   284.43
 27000.   150.84   284.75
 27900.   193.42   285.07
```

| | | |
|---|---|---|
| 28800. | 236.00 | 285.40 |
| 29700. | 284.40 | 285.85 |
| 30600. | 332.80 | 286.30 |
| 31500. | 381.20 | 286.75 |
| 32400. | 429.60 | 287.20 |
| 33300. | 474.60 | 287.40 |
| 34200. | 519.60 | 287.60 |
| 35100. | 564.60 | 287.80 |
| 36000. | 609.60 | 288.00 |
| 36900. | 640.00 | 288.12 |
| 37800. | 670.40 | 288.25 |
| 38700. | 700.80 | 288.37 |
| 39600. | 731.20 | 288.50 |
| 40500. | 749.20 | 288.60 |
| 41400. | 767.20 | 288.70 |
| 42300. | 785.20 | 288.80 |
| 43200. | 803.20 | 288.90 |
| 44100. | 764.60 | 288.85 |
| 45000. | 726.00 | 288.80 |
| 45900. | 687.40 | 288.75 |
| 46800. | 648.80 | 288.70 |
| 47700. | 646.20 | 288.70 |
| 48600. | 643.60 | 288.70 |
| 49500. | 641.00 | 288.70 |
| 50400. | 638.40 | 288.70 |
| 51300. | 594.40 | 288.70 |
| 52200. | 550.40 | 288.67 |
| 53100. | 506.40 | 288.65 |
| 54000 | 462.40 | 288.62 |
| 54900. | 419.60 | 288.60 |
| 55800. | 376.80 | 288.57 |
| 56700. | 334.00 | 288.55 |
| 57600. | 291.20 | 288.52 |
| 58500. | 250.80 | 288.50 |
| 59400. | 210.40 | 288.32 |
| 60300. | 170.00 | 288.15 |
| 61200. | 129.60 | 287.97 |
| 62100. | 102.80 | 287.80 |
| 63000. | 76.00 | 287.62 |
| 63900. | 49.20 | 287.45 |
| 64800. | 22.40 | 287.27 |
| 65700. | 0.0 | 287.10 |
| 66600. | 0.0 | 286.98 |
| 67500. | 0.0 | 286.85 |
| 68400. | 0.0 | 286.73 |
| 69300. | 0.0 | 286.60 |
| 70200. | 0.0 | 286.45 |
| 71100. | 0.0 | 286.30 |
| 72000. | 0.0 | 286.15 |
| 72900. | 0.0 | 286.00 |
| 73800. | 0.0 | 285.85 |
| 74700. | 0.0 | 285.70 |
| 75600. | 0.0 | 285.55 |
| 76500. | 0.0 | 285.40 |
| 77400. | 0.0 | 285.10 |
| 78300. | 0.0 | 284.80 |

| | | |
|---|---|---|
| 79200. | 0.0 | 284.50 |
| 80100. | 0.0 | 284.20 |
| 81000. | 0.0 | 284.10 |
| 81900. | 0.0 | 284.00 |
| 82800. | 0.0 | 283.90 |
| 83700. | 0.0 | 283.80 |
| 84600. | 0.0 | 283.45 |
| 85500. | 0.0 | 283.10 |
| 86400. | 0.0 | 282.4 |

# APPENDIX E

## Optimization program for System#2

```
C --------------------------------------------------
C    10/6/89
C --------------------------------------------------
C IN THIS PROGRAM THERE ARE TWO STATE VARIABLES Y1,Y2
C Y1 ---- STORAGE TANK TEMPERATURE (DEGREE KELVIN)
C Y2 ---- AMOUNT OF WATER AT A GIVEN STAGE (K)
C K  ----  REFERS TO STAGE -- TIME
C J  ----  REFERS TO STATES
C M1 ----  REFERS TO DIFFERENT FLOW RATE VALUES
C M2 ----  REFERS TO DIFFERENT AUXILIARY ENERGY VALUES
C DELT --  TIME STEP IN SECS
C UO(M1)--  FLOW RATE (LITERS/S)
C Y1K ---  VALUE OF THE STATE VARIABLE AT THE NEXT STAGE
C FR = 0.88 (APPROXIMATELY)
C As = SURFACE AREA OF THE STORAGE TANK
C Qs = SOLAR RADIATION
C Ta = AMBIENT TEMPERATURE (K)
C Ms = MASS OF STORAGE TANK (KG)
C SCK = ENTROPY GENERATION IN THE COLLECTOR
C SSK = ENTROPY GENERATION IN THE STORAGE TANK
C SMK = ENTROPY GENERATION IN THE MIXING VALVE
C--------------------------------------------------
C PART 2 OF THE PROGRAM : TRACING OF OPTIMAL TRAJECTORY
C UUO(K) -- OPTIMAL FLOW RATE AT K STAGE
C WWE(K) -- OPTIMAL AUXILIARY ENERGY AT STAGE K
C LL(K) --- TOTAL ENTROPY AT THE END OF STAGE K
C I(J1,J2,KMAX), UM(J1,J2,KMAX), WM(J1,J2,KMAX), Y1(J1),Y2(J2),UO(M),
C WE(M2), L(M1,M2), TI(M1,M2),UT(M1),UTN(M1),YP(J1)
C SC(J1,J2),SS(J1,J2),SM(J1,J2),SW(J1,J2)
C UUO(K),WWE(K),UUT(K)
C--------------------------------------------------
C
      IMPLICIT REAL*8(A-H, O-Z)
      REAL *8 MS, MC, I(61,51,97), UM(61,51,97),
     &     L(51), TI(51), IJKK1, IJKK2, IJK,IK, LL(97)
      DIMENSION TT(97), QS(97), TA(97), QS1(97),
     &        Y1(61), Y2(51), UO(51), WE(51),
     &        SC(61,51), SS(61,51), SM(61,51),SW(61,51),
     &        UT(51), YP(61), UTN(51),
     &        UUO(97), WWE(97), UUT(97)
C--------------------------------------------------
C          READING AND INITIALIZATION
C--------------------------------------------------
      KMAX = 97
      J1MAX = 61
      J2MAX = 51
      M1MAX = 51
C-----------------------
      DO 1 J1 = 1,J1MAX
      CJ1 = J1
    1 Y1(J1) = 310. + (CJ1-1.)
```

```
C-----------------------
      DO 2 J2 = 1,J2MAX
      CJ2 = J2
    2 Y2(J2) = (CJ2 -1.0)*5.0
C-----------------------
      DO 3 M1 = 1,M1MAX
      CM1 = M1
    3 UO(M1) = ((CM1 -1.0)*20.0)/3600.
C-----------------------
      DO 5 J1 = 1,J1MAX
      DO 5 J2 = 1,J2MAX
      UM(J1,J2,97) = 0.0
    5 CONTINUE
C-----------------------
      DO 6 K  = 1,KMAX
      DO 6 J1 = 1,J1MAX
      DO 6 J2 = 1,J2MAX
    6 I(J1,J2,K) = 1.0E+50
C----------------------------------------------------------
C INITIALIZATION OF COST AT THE BOUNDARY CONDITIONS
C FOR ANY CHANGE IN J1 &J2 VALUES, CHANGE THE SUBSCRIPTS OF I
C FOR ANY CHANGE IN TERMINAL TS, CHANGE THE SUBSCRIPTS  OF I
C----------------------------------------------------------
      I(17,50,97) = 1.0E+40
      I(17,51,97) = 1.0E+40
      I(18,50,97) = 1.0E+20
      I(18,51,97) = 1.0E+20
      I(19,50,97) = 1.0E+5
      I(19,51,97) = 0.0
      I(20,50,97) = 1.0E+20
      I(20,51,97) = 1.0E+20
      I(21,50,97) = 1.0E+40
      I(21,51,97) = 1.0E+40
C-----------------------
      DO 7 K = 1,97
      READ(5,*)TT(K),QS(K),TA(K)
    7 CONTINUE
C-----------------------
      AC=5.00
      CP=4190.
C     MS=450.
C     MS=350.
      MS=250.
C----------
      CS=4190.
      UL=4.
      US=0.5
C     AS=3.52
C     AS=2.9786
      AS=2.38
C----------
      TL = 273.+20.
      TM = 273.+20.
      TD = 273.+60.
C----------------------------------------------------------
C  FR FOR CONSTANT COLLECTOR FLOW RATE
```

```
C-----------------------------------------------------
C   MC=0.10
C   FR1=MC*CP/(AC*UL)
C   FR = FR1*(1.-DEXP(-0.9/FR1))
C-----------------------------------------------------
      DO 10 K = KMAX-1, 1, -1
      WRITE(15,*)K
       DO 15 J1 = 1, J1MAX
         QS1(K) = QS(K) -UL*(Y1(J1)-TA(K))
         IF(QS1(K) .LE. 0.0) THEN
         QS1(K) = 0.0
         MC = 0.
         FR = 0.01
         GOTO 602
         ELSEIF(QS1(K) .GT. 0.0)THEN
C-----------------------------------------------------
C     EXPRESSION FOR OPTIMAL MASS FLOW RATE THROUGH THE COLLECTOR
C-----------------------------------------------------
      FP  = 0.9
      AL  = 20.0
      D   = 0.01
      RHO = 1000.0
      F   = 0.03
      ACC = 2.5
      CP  = 4190.
      CF  =8.*F*AL/( (RHO**2)*(3.14**2)*D**5 )
      MC =2.* ( (ACC**2*UL*FP**2*QS1(K)*(Y1(J1)-TA(K))
     &            /(6.0*CF*Y1(J1)*CP) )**0.25
      FR1=MC*CP/(AC*UL)
      FR = FR1*(1.-DEXP(-0.9/FR1))
      ENDIF
C-----------------------------------------------------
C   RECTANGULAR GRID IS CONVERTED IN A TRIANGULAR ONE SO AS TO
C   CARRY OUT OPTIMIZATION DURING SOLAR INSOLATION.
C   THIS REDUCTION WAS DONE SINCE AFTER MANY OPTIMIZATION RUNS
C   IT WAS FOUND THAT ENTROPY GENERATION CONCENTRATION WAS HIGH
C   DURING SOLAR INSOLATION PERIOD
C-----------------------------------------------------
  602 IF(K .GE. 60)THEN
      J2UP = J2MAX
      J2DN = J2MAX - 6
      ELSEIF(K.LT.60 .AND. K .GE. 48)THEN
      J2UP = J2MAX
      J2DN = J2DN-4
      IF(J2DN .LE. 1)J2DN = 1
      ELSEIF(K .LT. 48 .AND. K .GE. 36)THEN
      J2UP = J2UP - 4
      J2DN = 1
      IF(J2UP .LE. 5) J2UP = 5
      ELSEIF(K .LT. 36)THEN
      J2UP = 5
      J2DN = 1
      ENDIF
C-----------------------------------------------------
C   L(M1) -- SINGLE STATE COST FUNCTION AT A GIVEN POINT
C   TI(M1) -- TOTAL COST AT A GIVEN POINT
```

```
C-----------------------------------------------
      DO 20 J2 = J2DN, J2UP
C---------
      DO 9 M1=1,M1MAX
 9    TI(M1) = 1.0E+50
C---------
      DO 40 M1 = 1,M1MAX
      UT(M1) = UO(M1)
      Y1K = Y1(J1) + (DELT/(MS*CS))*( AC*FR*QS1(K)
     &       - AS*US*(Y1(J1)-TA(K)) - UT(M1)*CP*(Y1(J1)-TL) )
      Y2K = Y2(J2) + UO(M1)*DELT
      IF( Y1K .LT. Y1(1) .OR. Y1K .GT. Y1(J1MAX) ) THEN
      TI(M1) = 1.0E+50
      GOTO 40
      ENDIF
      IF( Y2K .LT. Y2(J2DN) .OR. Y2K .GT. Y2(J2MAX) ) THEN
C     IF( Y2K .LT. Y2(J2DN) .OR. Y2K .GT. Y2(J2UP) ) THEN
      TI(M1) = 1.0E+50
      GOTO 40
      ENDIF
      IF( UO(M1) .LE. 0.0)THEN
      UO(M1) = 0.0
      WE(M1) = 0.0
      GOTO 43
      ELSEIF( (UO(M1) .GT. 0.0) .AND. (Y1(J1).LT.333.) ) THEN
      WE(M1) = UO(M1)*CP*(333.-Y1(J1) )
      GOTO 43
      ELSEIF( (UO(M1) .GT. 0.0) .AND. (Y1(J1).GE.333.) ) THEN
      GOTO 42
      ENDIF
C---------------------------
C      MIXING --DUE TO INCREASE IN TANK TEMP BEYOND 273.+60.
C      SM(J)---> MIXING ENTROPY
C-----------------------------------------------
 42   UT(M1) = UO(M1)*(333.-293.)/(Y1(J1)-293.)
      WE(M1) = 0.0
C---------------
C     INTERPOLATION
C---------------
C     DO 45 JJ2 = 1,J2MAX-1
C     IF( (Y2K .GE. Y2(JJ2)) .AND. (Y2K .LE. Y2(JJ2+1)) ) THEN
      DO 45 JJ2 = 1,J2MAX
      IF( (Y2K .GE. Y2(JJ2)-.1) .AND. (Y2K .LE. Y2(JJ2)+.1) ) THEN
      DO 46 JJ1 = 1,J1MAX-1
      IF( (Y1K .GE. Y1(JJ1)) .AND. (Y1K .LT. Y1(JJ1+1)) ) THEN
      JK1 = JJ1
      JK2 = JJ2
      IJK = I(JK1,JK2,K+1) + ( I(JK1+1,JK2,K+1)-I(JK1,JK2,K+1) )*
     &       (Y1K-Y1(JJ1))/(Y1(JJ1+1)-Y1(JJ1))
      GOTO 200
      ENDIF
 46   CONTINUE
      ENDIF
 45   CONTINUE
C-----------------------------------------------
200   SM(J1,J2) = DELT* ( UT(M1)*CP*DLOG(333./Y1(J1))
```

```
&      + (UO(M1)-UT(M1))*CP*DLOG(333./TL) )
      SW(J1,J2) = 0.0
C-----------------------------------------------
C 43  DO 47 JJ2 = 1,J2MAX-1
C     IF( (Y2K .GE. Y2(JJ2)) .AND. (Y2K .LE. Y2(JJ2+1)) ) THEN
C-----------------------------------------------
C     THERE IS SUBTRACTION AND ADDITION OF 0.1 IN THE ABOVE IF
C     STATEMENT, THIS IS DONE TO FACILITATE NUMERICAL ANALYSIS
C-----------------------------------------------
 43   DO 47 JJ2 = 1,J2MAX
      IF( (Y2K .GE. Y2(JJ2)-.1) .AND. (Y2K .LE. Y2(JJ2)+.1) ) THEN
      DO 48 JJ1 = 1,J1MAX-1
      IF( (Y1K .GE. Y1(JJ1)) .AND. (Y1K .LT. Y1(JJ1+1)) ) THEN
      JK1 = JJ1
      JK2 = JJ2
      IJK = I(JK1,JK2,K+1) + ( I(JK1+1,JK2,K+1)-I(JK1,JK2,K+1) )*
     &       (Y1K-Y1(JJ1))/(Y1(JJ1+1)-Y1(JJ1))
      GOTO 300
      ENDIF
  48  CONTINUE
      ENDIF
  47  CONTINUE
C-----------------------------------------------
 300  SM(J1,J2) = 0.0
      SW(J1,J2) = DELT*UO(M1)*CP*DLOG(333./Y1(J1))
C-----------------------------------------------
C       ENTROPY (COST FUNCTION)
C-----------------------------------------------
 100  IF(QS1(K) .LE. 0.0)THEN
      YP(J1) = TA(K) +QS(K)/UL
      ELSEIF(QS1(K) .GT. 0.0) THEN
      YP(J1) = Y1(J1) + QS1(K)*(1.-FR)/UL
      ENDIF
      SC1 = DELT*(-QS(K)*AC/YP(J1))
      SC2 = DELT*(+AC*UL*(YP(J1)-TA(K))/TA(K))
      SC(J1,J2)  = SC1 +SC2
C-----------------------------------------------
      SS1 = DELT*(+ AC*FR*QS1(K)/Y1(J1))
      SS2 = DELT*(- US*AS*(Y1(J1)-TA(K))/Y1(J1))
      SS3 = DELT*(+UT(M1)*CP*(TL-Y1(J1))/Y1(J1))
      SS5 = DELT*(+ AS*US*(Y1(J1)-TA(K))/TA(K))
      SS6 = DELT*(+UT(M1)*CP*DLOG(Y1(J1)/TL))
      SS(J1,J2) = SS1+SS2+SS3+SS5+SS6
      L(M1) = SC(J1,J2) +SS(J1,J2) +SM(J1,J2) +SW(J1,J2)
      TI(M1) = L(M1) + IJK
  40  CONTINUE
C-----------------------------------------------
C     FINDING MINIMUM ENTROPY AND ASSOCIATED OPTIMUM UO AND WE
C-----------------------------------------------
      I(J1,J2,K) = TI(1)
      UM(J1,J2,K)= UO(1)
      DO 33 M1 =2,M1MAX
      IF(TI(M1) .LT. I(J1,J2,K)) THEN
      I(J1,J2,K)=TI(M1)
      UM(J1,J2,K)= UO(M1)
      ENDIF
```

```
   33 CONTINUE
C--------------------------------------------------
   20 CONTINUE
   15 CONTINUE
   10 CONTINUE
C--------------------------------------------------
C      PRINTING OF BACKWARD OPTIMUM RESULTS
C        (DYNAMIC PROGRAMMING SOLUTION)
C--------------------------------------------------
      DO 49 K = KMAX,1,-1
C   IF(K .EQ. 1 .OR. K .EQ. 95 .OR. K .EQ. 96)THEN
      IF(K .EQ. 1)THEN
      WRITE(6,900)K
  900 FORMAT('    K = ',I2)
      WRITE(6,910)( J1,J1=1,10)
  910 FORMAT(10X,10I11)
      DO 50 J2 = 1,J2MAX
      WRITE(6,920)Y2(J2),(I(J1,J2,K),J1=1,10)
  920 FORMAT(11E11.4)
      WRITE(6,922)(3600.*UM(J1,J2,K),J1=1,10)
  922 FORMAT(11X,10E11.4)
      WRITE(6,925)
  925 FORMAT(' ')
   50 CONTINUE
C-----
      WRITE(6,911)(J1,J1=11,20)
  911 FORMAT(10X,10I11)
      DO 51 J2 = 1,J2MAX
      WRITE(6,912)Y2(J2),(I(J1,J2,K),J1=11,20)
  912 FORMAT(11E11.4)
      WRITE(6,913)(3600.*UM(J1,J2,K),J1=11,20)
  913 FORMAT(11X,10E11.4)
      WRITE(6,930)
  930 FORMAT(' ')
   51 CONTINUE
C-----
      WRITE(6,915)(J1,J1=21,30)
  915 FORMAT(10X,10I11)
      DO 52 J2 = 1,J2MAX
      WRITE(6,916)Y2(J2),(I(J1,J2,K),J1=21,30)
  916 FORMAT(11E11.4)
      WRITE(6,917)(3600.*UM(J1,J2,K),J1=21,30)
  917 FORMAT(11X,10E11.4)
      WRITE(6,935)
  935 FORMAT(' ')
   52 CONTINUE
C--------
      WRITE(6,951)(J1,J1=31,41)
  951 FORMAT(10X,11I11)
      DO 53 J2 = 1,J2MAX
      WRITE(6,952)Y2(J2),(I(J1,J2,K),J1=31,41)
  952 FORMAT(12E11.4)
      WRITE(6,953)(3600.*UM(J1,J2,K),J1=31,41)
  953 FORMAT(11X,11E11.4)
      WRITE(6,954)
  954 FORMAT(' ')
```

```
 53 CONTINUE
C-------
    ENDIF
 49 CONTINUE
C-----------------------------------------------
C   TRACING OPTIMAL TRAJECTORY AND MINIMUM ENTROPY
C-----------------------------------------------
C   INITIALIZATION
C-----------------------------------------------
    WRITE(6,888)
 888 FORMAT(//8X,' Y1K    Y2K    UUT    WWE   IK   WK '//)
    Y1K = 333.
    Y2K = 0.0
    IK = 0.0
    WK = 0.0
C-----------------------------------------------
    DO 71 K = 1,KMAX-1
C   IF( Y1K.LT.Y1(1) )   Y1K = Y1(1)
C   IF( Y1K.GT.Y1(J1MAX) ) Y1K = Y1(J1MAX)
C   IF( Y2K.LT.Y2(1) )   Y2K = Y2(1)
C   IF( Y2K.GT.Y2(J2MAX) ) Y2K = Y2(J2MAX)
    DO 72 JJ2 = 1,J2MAX-1
    IF( (Y2K .GE. Y2(JJ2)) .AND. (Y2K .LE. Y2(JJ2+1)) ) THEN
    DO 73 JJ1 = 1,J1MAX-1
    IF( (Y1K .GE. Y1(JJ1)) .AND. (Y1K .LT. Y1(JJ1+1)) ) THEN
    JK1 = JJ1
    JK2 = JJ2
C-----------------------------------------------
C   THERE IS SUBTRACTION AND ADDITION OF 0.1 IN THE ABOVE IF
C   STATEMENT, THIS IS DONE TO FACILITATE NUMERICAL ANALYSIS
C-----------------------------------------------
C   IF( (Y2K.GE. Y2(J2MAX)-.1) .AND. (Y2K.LE. Y2(J2MAX)+.1) )
C  & JK2 = J2MAX
C-----------------------------------------------
C    TWO DIMENSIONAL INTERPOLATION
C-----------------------------------------------
    UJKK1 =UM(JK1,JK2,K)+(UM(JK1,JK2+1,K)-UM(JK1,JK2,K) )*
   &   (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
    UJKK2 = UM(JK1+1,JK2,K)+(UM(JK1+1,JK2+1,K)-UM(JK1+1,JK2,K))*
   &   (Y2K-Y2(JJ2))/(Y2(JJ2+1)-Y2(JJ2))
    UUO(K) = UJKK1 + (UJKK2-UJKK1)*(Y1K-Y1(JJ1))/
   &    ( Y1(JJ1+1)-Y1(JJ1) )
C-----------------------------------------------
C    ONE DIMENSIONAL INTERPOLATION
C-----------------------------------------------
C   UUO(K) = UM(JK1,JK2,K)+(UM(JK1+1,JK2,K)-UM(JK1,JK2,K))*
C  &   (Y1K-Y1(JJ1))/(Y1(JJ1+1)-Y1(JJ1))
C-----------------------------------------------
    GOTO 74
    ENDIF
 73 CONTINUE
    ENDIF
 72 CONTINUE
 74 IF( UUO(K) .LE. 0.0 ) THEN
    UUT(K) = 0.0
    WWE(K) = 0.0
```

```
      GOTO 76
      ELSEIF( (UUO(K).GT. 0.0) .AND. (Y1K.LT.333.) )THEN
      WWE(K) = UUO(K)*CP*(333.-Y1K)
      UUT(K) = UUO(K)
      GOTO 76
      ELSEIF( (UUO(K).GT. 0.0) .AND. (Y1K.GE.333.) )THEN
      GOTO 75
      ENDIF
C-----------------------------------------
  75 UUT(K) = UUO(K)*(333.-293.)/(Y1K-293.)
      SMK= DELT* (UUT(K)*CP*DLOG(333./Y1K)
     &      + (UUO(K)-UUT(K))*CP*DLOG(333./TL) )
      WWE(K)=0.0
      SWK = 0.0
      GOTO 500
C-----------------------------
  76 SWK = DELT*UUT(K)*CP*DLOG(333./Y1K)
      SMK=0.0
C-----------------------------
 500 QS1(K) = QS(K) -UL*(Y1K-TA(K))
C-----------------------------------------
C      EXPRESSION FOR OPTIMAL MASS FLOW RATE THROUGH THE COLLECTOR
C-----------------------------------------
      IF(QS1(K) .LE. 0.0)THEN
      YPK = TA(K) +QS(K)/UL
      QS1(K) = 0.0
      MC = 0.
      FR = 0.01
      GOTO 603
      ELSEIF(QS1(K) .GT. 0.0)THEN
      FP  = 0.9
      AL  = 20.0
      D   = 0.01
      RHO = 1000.0
      F   = 0.03
      ACC = 2.5
      CP  = 4190.
      CF  =8.*F*AL/( (RHO**2)*(3.14**2)*D**5 )
      MC =2.* ( (ACC**2*UL*FP**2*QS1(K)*(Y1K-TA(K)))
     &              /(6.0*CF*Y1K*CP) )**0.25
      FR1=MC*CP/(AC*UL)
      FR = FR1*(1.-DEXP(-0.9/FR1))
C-----------------------------------------
      YPK = Y1K + QS1(K)*(1.-FR)/UL
      ENDIF
C--------
 603 SC1 = DELT*(-QS(K)*AC/YPK)
      SC2 = DELT*(+AC*UL*(YPK-TA(K))/TA(K))
      SCK = SC1 +SC2
C-----------------------------------------
      SS1 = DELT*(+ AC*FR*QS1(K)/Y1K)
      SS2 = DELT*(- US*AS*(Y1K-TA(K))/Y1K)
      SS3 = DELT*(+UUT(K)*CP*(TL-Y1K)/Y1K)
      SS5 = DELT*(+ AS*US*(Y1K-TA(K))/TA(K))
      SS6 = DELT*(+UUT(K)*CP*DLOG(Y1K/TL) )
      SSK = SS1+SS2+SS3+SS5+SS6
```

```
      LL(K) = SCK +SSK +SMK +SWK
      IK = IK + LL(K)
      WK = WK + WWE(K)
      WRITE(6,899)K,Y1K,Y2K,3600*UUO(K),WWE(K),IK,WK
  899 FORMAT(2X,I4,6E11.4)
      Y1K =Y1K+(DELT/(MS*CS))*( AC*FR*QS1(K)
     &    -AS*US*(Y1K-TA(K))-UUT(K)*CP*(Y1K-TL) )
      Y2K = Y2K +UUO(K)*DELT
      IF(K.EQ.96)THEN
      WRITE(6,950)Y1K,Y2K
  950 FORMAT(5X,2E12.4)
      ENDIF
   71 CONTINUE
      STOP
      END
```